$y = f(x) = x^2$

1. translate 4 units to the left
2. " 5 " up

a) $g(x) = ?$

no horizontal stretch $\Rightarrow b = 1$

$\Rightarrow$ up by 5 $\Rightarrow k = 5$

The general form is:

$$y = af\left[b(x-h)\right] + k$$

Left by 4 $\Rightarrow (x+4)$

$(x-(-4)) \Rightarrow h = -4$

no horizontal transformation

$\Downarrow$

$a = 1$

$\therefore$ the transformation is $g(x) = f(x+4) + 5$

the equation of $g(x)$ is then

$$\begin{aligned} g(x) &= (x+4)^2 + 5 \\ &= x^2 + 2 \cdot 4 \cdot x + 16 + 5 \\ &= x^2 + 8x + 21 \end{aligned}$$

$\Downarrow$

$g(x) = x^2 + 8x + 21$

b) First: let's clarify what is the image function. It's is the function we are mapping to, which in this case is: $g(x)$.

There is nothing in this problem to limit the domain of $g(x)$

$$\Rightarrow D: x \in \mathbb{R}$$

if you read the question in section (c) you will see that

if you read the question in
section (c) you will see that
there, you are required to interpret the translations from $f(x)$ to $g(x)$ visually.
so we look for another way to answer the question in this section (b).

The range of $g(x)$ depends on the values that it can reach.
The coefficient of x^2 in $g(x)$ is $1 \Rightarrow$ this
parabola has its arms up. This means the
parabola has a minimum vertex, and then

all its y values are larger or equal to its y of the vertex.

Let's calculate the vertex by expressing $g(x)$
into vertex form. We use "complete the square"
for this.

$$g(x) = x^2 + 8x + 21$$
$$= x^2 + 2\cdot 4x + 4^2 - 4^2 + 21$$
$$= (x+4)^2 - 16 + 21$$
$$= (x+4)^2 + 5$$

$\equiv$ to the template $(x-p)^2 + h$
where th vertex is
$V(p,h)$

$$= (x-(-4))^2 + 5$$
$$\Downarrow$$
$$V(-4, 5)$$

x_v

$y_{v=5}$ ← all y-s are $\geq y_v$

$$\Downarrow$$

The range is $\{y \mid y \geq 5, y \in \mathbb{R}\}$

c)
step1:
a shift of 4 units to the left
this transformation has no effect on
the domain or the range.

$f(x) = x^2$ → domain $x \in \mathbb{R}$
→ range $y \geq \emptyset, y \in \mathbb{R}$

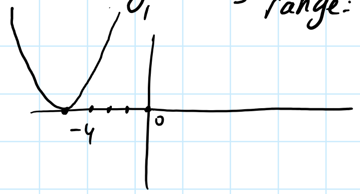
← $y = \emptyset$

then $g(x)$ also remains such:
$g(x)$ → domain : $x \in \mathbb{R}$

then $g_1(x)$ also remains such:

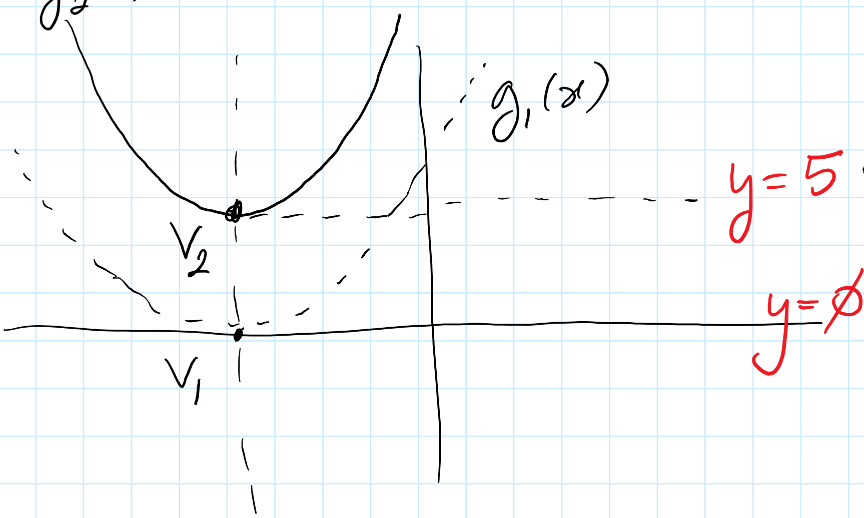
$g_1(x)$ domain: $x \in \mathbb{R}$

range: $y \in \mathbb{R}, y \geq \emptyset$.



Step2:

$g_2(x)$ = a shift up by 5 units.



$g_2(x)$: now everything is higher, all y values are above the translated vertex the original vertex was 0, the shifted x vertex moved up by 5 unit => the entire range moved up by 5 units as well.

->//