p 015 q17

Extend

17. The graph of the function $y = x^2$ is translated to an image parabola with zeros 7 and 1.

a) Determine the equation of the image function.

b) Describe the translations on the graph of $y = x^2$.

c) Determine the y-intercept of the translated function.

translation = shift

said translated
not said t b stretched

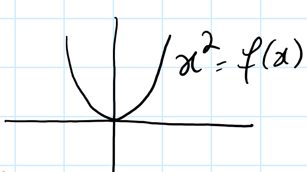
(a) roots given as 1 & 7, we name them α & β.

$1 \to \alpha$, $7 \to \beta$

if α & β are roots then they factor the function

$$f(x) = a(x-\alpha)\cdot(x-\beta)$$
$$= a(x-1)(x-7)$$

The problem states the function was only translated. This can be horizontally and/or vertically. There are no stretches.



(this has only one point of contact w/ the x-axis...)

→ to have these two roots the function had to move to the right and down.

↳ If the function did not move down, it would not be able to intersect the x-axis in 2 distinct points.

Since no stretches were indicated we can conclude that

a & b in the formula $g(x) = af(b(x-h)) + k$

are both 1 : $a = b = 1$, a.k.a they are not applicable.

This means $g(x)$ is simpler: $g(x) = 1 \cdot f(1 \cdot (x-h)) + k$

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Therefore $g(x) = f(x-h) + k = (x-h)^2 + k$

& also $g(x) = a(x-1)(x-7) = (x-1)(x-7)$

for the same function $g(x)$

then we can equate them.

$\therefore \quad g(x) = \boxed{(x-h)^2 + k = (x-1)(x-7)}$

$(x-1)(x-7) = x^2 - 7x - x + 7$
$= x^2 - 8x + 7$

we do not need to find k, we only need the equation of the g(x)

To answer (a), it is enough to state $g(x) = x^2 - 8x + 7$ is the equation of the image function.

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we already know from (a) the function had only translations, no stretches.

b) $g(x) = \underbrace{x^2 - 8x + 7}_{\circledast} = \underbrace{(x-h)^2 + k}$

finding h & k tells us about the vertical/horizontal translations that happened to this function

this is a transformation form.

this form is not organized in a way that shows the transformations.

this form is also equivalent to vertex-form which we can obtain from $\circledast$ by completing the square. If we calculate the vertex-form, we get the equivalent transformation form.

complete the square

$x^2 - 8x + 7 = x^2 - 2 \cdot 4x + 7 = x^2 - 2 \cdot 4x + \overbrace{4^2 - 4^2}^{\varnothing} + 7 =$

assign to the left and make a square

assign to the right

the square part

$$= \left(x^2 - 2\cdot 4x + 4^2\right) - 4^2 + 7 =$$

$$= (x-4)^2 - 16 + 7$$

$$= (x-4)^2 - 9$$

$$= (x-4)^2 + (-9) = \text{vertex form}$$

the vertex is $V(4, -9)$

"$(x-p)^2 + q$"

vertex form comes w/ a plus...

Vertex = $V(p, q)$

also = horizontal shift

also = vertical shift

$$g(x) = (x-4)^2 + (-9)$$

& since $f(x) = x^2$ it's easy to see $(x-4)^2 = f(x-4)$

b/c: $f(\square) = \square^2$

$$\Rightarrow g(x) = f(x-4) - 9$$

the transformations are

→ horizontal shift: 4 units to the right

→ vertical shift: 9 units downwards.

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c) $f(x) = (x-1)(x-7)$

$f(0) = (0-1)(0-7)$

$f(0) = (-1)(-7)$

$f(0) = 7$ = the y-intercept
