

- C1** The graph of the function $y = f(x)$ is transformed to the graph of $y = f(x - h) + k$.
- a) Show that the order in which you apply translations does not matter. Explain why this is true.
- b) How are the domain and range affected by the parameters h and k ?

a)

$f(x)$ = parent base
 $g(x) = f(x - h) + k$ → shift up by k units.
 ↓
 shift to the right by h units

the mapping: $(x, y) \rightarrow (x + h, y + k)$

you can calculate the new x first & has no effect on the new y .

you can calculate the new y first too, and it has no effect on the x calculation

the order will not matter since the two different dimensions x and y do not affect each other.

* Another way to prove this is algebraic:

$$g(x) = f(x - h) + k$$

let's take an example parent: $f(x) = x^2$

then { Horiz. Sh. 1st: $g_1(x) = f(x - h) = (x - h)^2$

{ V. Sh. 2nd: $g_2(x) = g_1(x) + k = (x - h)^2 + k = g(x)$

Now try the reverse order { V. Sh. 1st: $g_1(x) = f(x) + k = x^2 + k$

{ H. Sh. 2nd: $g_2(x) = g_1(x - h) = (x - h)^2 + k = g(x)$

$$= (x-h) + k$$

$$= g(x)$$

In both orders we get $g(x) = (x-h)^2 + k$
 $\therefore$ the order did not matter.

b) Domain & Range

$$g(x) = f(x-h) + k$$

The domain of $f(x)$ has 2 possibilities

- restricted (case ii)
- unrestricted (case i)

Case i

shifting an unrestricted domain: i.e. $x \in \mathbb{R}$
 left or right has no effect. It remains $x \in \mathbb{R}$ } has no effect on the range & vice versa.

Case ii

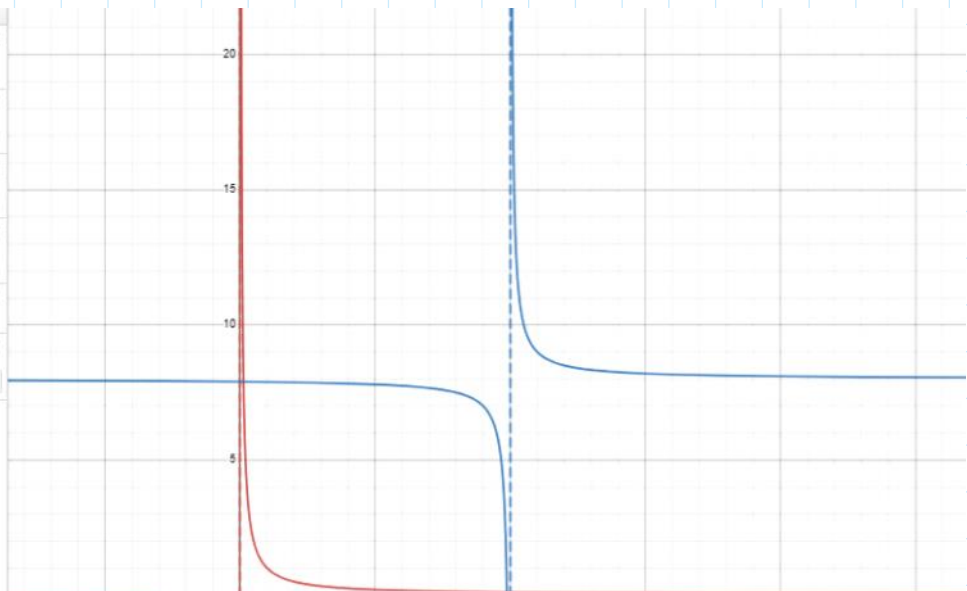
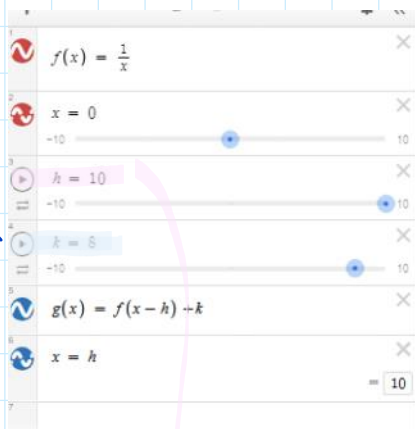
if the domain is restricted, let's say for ex.
 by an asymptote, then the asymptote is
 moved in the direction of the horizontal shift.

But this is not influenced by the vertical
 shift "+k".

For example visually, playing w/ the Vertical Shift parameter k
 below has no effect on the domain.

Run the play button to see this is so.

<https://www.desmos.com/calculator/ptm7qzrqka>



alternately

alternately
playing w/ the
horizontal parameter h
(use the play button
to see) has no effect
on the Range.

