p 015 qC2

**C2** Complete the square and explain how to transform the graph of $y = x^2$ to the graph of each function.

a) $f(x) = x^2 + 2x + 1$

b) $g(x) = x^2 - 4x + 3$

a) complete the square for

$$f(x) = \underbrace{x^2 + 2x}_{\text{this has the start of a perfect square}} + 1 = \underbrace{\boxed{\quad\quad}^{2}}_{\text{a perfect square}} + \underbrace{\boxed{\ }}_{\text{an imperfection}}$$

this has the start of a perfect square: $x^2 + 2x + \dots = x^2 + 2\cdot 1 \cdot x + 1^2 + \dots$

it's not present, so we add it but then we must also remove it as follows.

give it to the perfect square

give it to the imperfection

$$x^2 + 2x + \dots = x^2 + 2\cdot 1\cdot x + \underbrace{1^2 - 1^2}_{\text{overall zero so it makes no differene}} + \dots = x^2 + 2x + 1 - 1 + \dots$$

$$= (x+1)^2 - 1 + \dots \qquad (*)$$

the original expression was

$$f(x) = \underbrace{x^2 + 2x}_{(*)} + 1 = \underbrace{(x+1)^2 - 1}_{} \underbrace{+ 1}_{-1+1=\emptyset} = (x+1)^2$$

the question asked you perform the "complete the square" the question asked you perform the "complete the square" method, but it was not really necessary to conclude that $x^2+2x+1=(x+1)^2$ since this is based on the algebraic formula

$$(a+b)^2=a^2+2ab+b^2$$

and $(a+1)^2=a^2+2a+1$

$\therefore \quad f(x)=(x+1)^2$

and the base parent function (let's name it $p(x)$) is $p(x)=x^2$

Explanation: $p(x)$ transforms to $f(x)$ based on:

$p(x)=\square^2$ uses $x$ $\qquad$ $f(x)=\square^2$ uses $x+1$

the difference: other than the parameter $x \to x+1$ the rest of the pattern of the functions is the same

$\therefore$ our mapping is:

$$p(x) \to f(x)$$

$$x \to x+1$$

The answer

$\boxed{\text{this is a horizontal shift to the left by 1 unit.}}$

1 unit.

b) $g(x) = x^2 - 4x + 3$

↓ "complete the square":

$g(x) = x^2 - 4x + 3$ ← "imperfection"

$= x^2 - 2 \cdot 2x + 3$

↳ would be a perfect square if u added $2^2$

like so:

$x^2 - 2 \cdot 2x + \underbrace{2^2}_{\text{but u cant just add it,}} = (x-2)^2$

so you must remove it as well

$+2^2 - 2^2 = \varnothing$

$$g(x) = \overbrace{x^2 - 2 \cdot 2x + 2^2}^{\text{forms a perfect square}} \underbrace{+2^2 - 2^2}_{\text{zero}} + 3$$

forms an imperfection

$= (x-2)^2 - 2^2 + 3$

$= (x-2)^2 - 4 + 3$

$\underbrace{g(x) = (x-2)^2 - 1}_{\text{from this form}}$, done w/ "complete the square"

we can conclude tranformations of the base parent $p(x) = x^2$

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<u>patterns</u>: $p(x) = \underbrace{\blacksquare}_{x}{}^2$ & $g(x) = \underbrace{\blacksquare}_{x-2}{}^2 - 1$

parameters:

$$\Rightarrow g(x) = p(x-2) - 1$$

This means a

The answer

$\boxed{\begin{array}{l} \text{1) Horizontal shift to the Right} \\ \quad \text{by 2 units, and} \\ \text{2) a Vertical shift down by 1 unit.} \end{array}}$