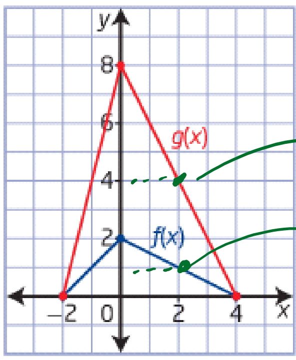
p 028, q7

7. Describe the transformation that must be applied to the graph of $f(x)$ to obtain the graph of $g(x)$. Then, determine the equation of $g(x)$ in the form $y = af(bx)$.

a)



$\to A'(2,4)$

$\to A(2,1)$

a) the domain has not changed. In both cases we have
$x \in [-2,4]$
therefore no horizontal changes have occured.
In the formula
$y = af(bx)$
b must be 1.
$\therefore y = af(x)$

There is a vertical stretch that has increased the range from
$y \in [0,2]$ in $f(x)$
to $y \in [0,8]$ in $g(x)$, possibly by a factor of 4 b/c $8 = 4 \cdot 2$. The mapping should be: $(x,y) \to (x, 4y)$

We can verify a point on $g(x)$ to see this is true

$A \to A'$

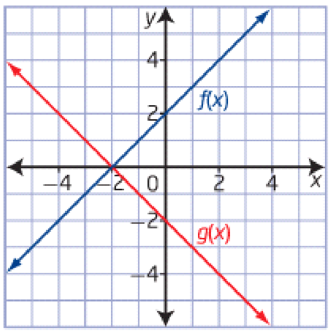
$A(2,1) \to A'(2,4)$

$(2,1) \to (2, 4 \cdot 1)$ ✓ this verifies

Answer: there was a vertical stretch by a

Answer: there was a vertical stretch by a factor of 4. $\Rightarrow \boxed{g(x) = 4f(x)}$

b)



The blue function $\overset{f(x)}{\vee}$ is perfectly symmetric to the red $g(x)$.

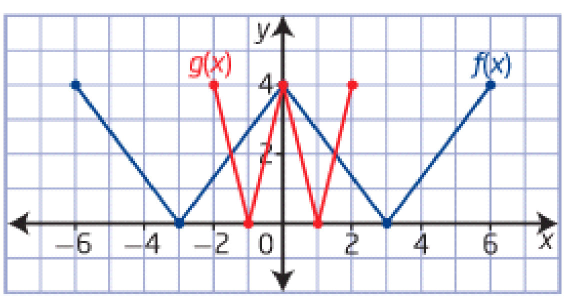
The symmetry is such that the x has not changed by the y-s were flipped across the x-axis.

Thus: $f \rightarrow g$

$$(x, y) \rightarrow (x, -y)$$

This describes a vertical reflection into the x-axis: $\boxed{g(x) = -f(x)}$

c)



$f \rightarrow g$

the Range is the same in both functions:

$$R_f = R_g = \{y \mid y \in [0, 4], y \in \mathbb{R}\}$$

thus the coefficient representing vertical changes in

$$y = af(bx)$$

must be ...

$y = a f(bx)$

must be 1:

$a = 1 \Rightarrow \boxed{y = f(bx)}$

thus no vertical transformations have happened. The domain on the other hand is stretch to be shorter

from $D_f = \{x \mid x \in [-6, 6], x \in \mathbb{R}\}$

to $D_g = \{x \mid x \in [-2, 2], x \in \mathbb{R}\}$

By analyzing the edges of these intervals we see that

$$\xrightarrow{f \to g}$$

left side: $(x, \ldots) \rightarrow (\frac{x}{3}, \ldots)$

$-6 \qquad \frac{-6}{3} = -2$

right side: $(x, \ldots) \rightarrow (\frac{x}{3}, \ldots)$

$+6 \qquad \frac{+6}{3} = 2$

In both case the horizontal stretch factor is $\frac{1}{3}$

This factor turns upside-down to $\frac{3}{1} = 3$ when used in the function formula instead of the mapping.

This means the horizontal coefficient b in the formula

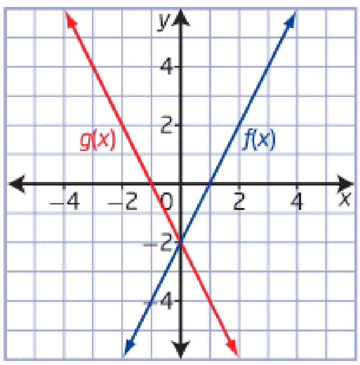
$y = f(bx)$

$y = f(6x)$

is $b = 3$.

$\therefore$ $\boxed{y = f(3x)}$

d)



In this example the x values were changed & the y values stayed the same b/c $f(x)$ & $g(x)$ are symmetric in relation to the y-axis.

The mapping based on what we see on this graph is:

$$f \rightarrow g$$
$$(x, y) \rightarrow (-x, y)$$

This describes a horizontal reflection into the y-axis, therefore the formula is

$$\boxed{g(x) = f(-x)}.$$