

9. Describe what happens to the graph of a function $y = f(x)$ after the following changes are made to its equation.

- a) Replace x with $4x$.
- b) Replace x with $\frac{1}{4}x$.
- c) Replace y with $2y$.
- d) Replace y with $\frac{1}{4}y$.
- e) Replace x with $-3x$.
- f) Replace y with $-\frac{1}{3}y$.

a) $x \rightarrow 4x \Rightarrow g(x) = f(4x)$

a horizontal stretch by a factor of $\frac{1}{4}$

b) $x \rightarrow \frac{1}{4}x \Rightarrow g(x) = f\left(\frac{1}{4}x\right)$

a horizontal stretch by a factor of 4

b/c $\frac{1}{\frac{1}{4}} = 1 \cdot \frac{4}{1} = 4$

c) $y = f(x) :$

↑ we change the equation (not the coordinates)

$2y = f(x) \Rightarrow y = \frac{1}{2}f(x)$

$\div 2$

This is a vertical stretch by a factor of $\frac{1}{2}$.

vertical stretch by a factor of $\frac{1}{2}$!

d) $y = f(x)$

replace w/ $\frac{1}{4}y$ in the equation.

we get:

$$\frac{1}{4}y = f(x) \quad | \quad (\times 4) \text{ to both sides}$$

$$y = 4f(x)$$

This is a vertical stretch by a factor of 4.

e) $y = f(x)$

replace w/ $-3x$

$$y = f(-3x)$$

this is horizontal stretch by a factor of $\frac{1}{3}$, and a horizontal

reflection into the y-axis. These two steps can be applied in any order.

f) $y = f(x)$

replace w/ $-\frac{1}{2}x$

$$-\frac{1}{3}y$$

$$-\frac{1}{3}y = f(x) \quad | \quad (x-3)$$

$$y = -3f(x)$$

This is a vertical stretch by a factor of 3 and a vertical reflection into the x -axis, the two steps can happen in any order & will have the same result.