p 031, qC1

C1 Explain why the graph of $g(x) = f(bx)$ is a horizontal stretch about the y-axis by a factor of $\frac{1}{b}$, for $b > 0$, rather than a factor of b.

Let's say we are looking for the horizontal mapping

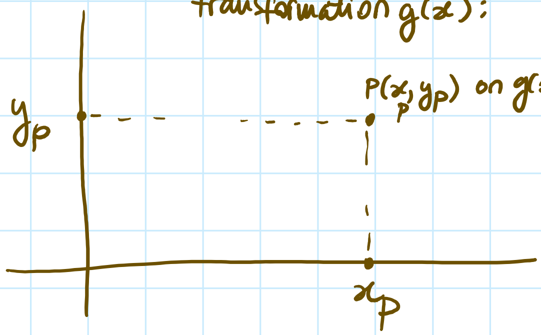
$g: (x, y) \to (t \cdot x, y)$

for the formula $g(x) = f(bx)$

and we want to proove that $\boxed{t = \frac{1}{b}}$.

The horizontal stretch factor about the y-axis is taken from the mapping, namely from the horizontal element of the mapping.

Proof: let's plot a point $P(x_p, y_p)$ on the transformation $g(x)$:



We can backtrack the point $P(x_p, y_p)$ from the transformation to the original function $g(x)$ based on:

general: $g(x) = f(bx)$

specifically for P: $g(x_p) = f(\underline{bx_p})$ → let's name this point x_A

$$g(x_p) = f(bx_p) = f(x_A)$$

You can word this as: $g(x)$ transports the x_A to x_p, i.e.

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$$x_A \xrightarrow[g(x)]{} x_p$$

but we can also say that

$$y_p = g(x_p) = f(bx_p) = f(x_A) = y_A \quad \text{(Expr. 1)}$$

So this means: $y_p = y_A$

different x-s but the same y. This is also b/c this is a horizontal only transformation, so there should be no changes to y.

From Expr. 1 we can extract the relationship between the x-s so we can construct our mapping.

we have: $g(x_p) = f(bx_p) = f(x_A)$

these 2 parameters must be the same x:

$$b x_p = x_A \quad \text{Expr 2}$$

We want to know how x_A goes to x_p, i.e. what does the mapping look like: $x_A \to x_p$ but this mapping should: all be a function of x_A, i.e. $f(x)$'s coordinates. For this we isolate x_p as a relationship of x_A:

from Exp2: $$x_p = \frac{x_A}{b}$$

$$x_p = \frac{1}{b} \cdot x_A$$

Then the full mapping $x_A \to x_P$ is

$$(x_A, y_A) \to \left(\frac{1}{b} x_A, y_P\right)$$

b/c $y_A = y_P$: $(x_A, y_A) \to \left(\frac{1}{b} x_A, y_A\right)$

after generalizing for all x: $(x, y) \to \left(\frac{1}{b} x, y\right)$

$\therefore$ the horizontal stretch factor of the transformation

$$g(x) = f(bx)$$

is $\frac{1}{b}$.

Q.E.D.