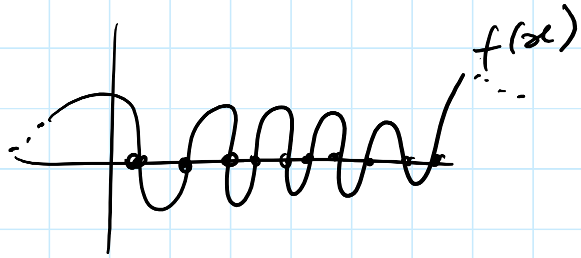
p 031, qC2

C2 Describe a transformation that results in each situation. Is there more than one possibility?

a) The x-intercepts are invariant points.

b) The y-intercepts are invariant points.

a) x-intercepts = the roots



We don't want the roots to change, i.e. keep them invariant.

An easy way to do so is to apply no horizontal transformations, and we must insure that whatever vertical transf. is used, does not affect the $y = \emptyset$ coordinate. Otherwise the roots are moved, so not invariant.

(i) multiplication is ok b/c $\emptyset \to \emptyset \cdot k = \emptyset$

$$(x, \emptyset) \to (x, \emptyset)$$

(ii) shift is not ok b/c $\emptyset \to \emptyset + t \neq \emptyset$ move the root to $(x, \emptyset) \to (x, t)$ which is not invariant.

∴ (i) vertical stretch is ok

(ii) " shift is not ok

$$(x, y) \to (x, k \cdot y)$$

is a good transformation for this, for any k.

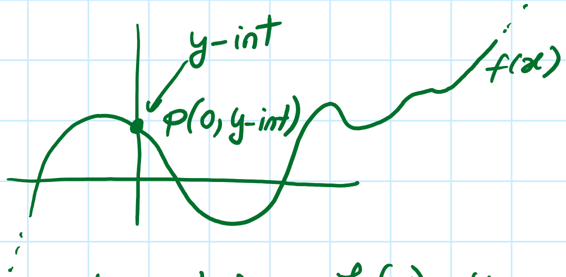
The answer:

$k \neq -1 \Rightarrow$ a Vert. stretch: $(x,y) \to (x, ky)$ would work,

$k = -1 \Rightarrow$ a Vert. reflection: $(x,y) \to (x,-y)$ also keeps the roots invariant.

So yes, there are more than just one option.

b) The y-int. to be invariant. Let's name the y-intercept point P.



A y-int. is $= f(\emptyset) = y_p$

If the transf. leaves all y-s still & only changes the x, points like P have their y coordinate unchanging = invariant, but the y-int. as a whole is invariant, i.e. a point such as $P(0, y_p)$ only if the transformation also makes no change to the x-coord specifically when x is zero.

If $\emptyset$ is moved $\underbrace{(0, y_p)}_{\text{the y-int.}} \to (x_Q, y_p)$

x_Q ↳ some non-zero value

than P is no longer invariant even if its y_p stayed the same. P must not reposition with transformation at all.

Here are two horizontal transform. examples (let's say $b \neq 0$ & $t \neq 0$)

1) $x \to x \times b$

2) $x \to x \times b + t$

Option 1) preserves $x=0$: $\emptyset \to \emptyset \cdot b = \emptyset$

Option 2) does not preserve the zero

$$\emptyset \to \emptyset \cdot b + t = \emptyset + t = t$$

Since t in Option 2 represents a horizontal shift, we must avoid it. But the horizontal stretch b is just fine.

$\therefore$ The transf. $(x, y) \to (bx, y)$ preserves the y-intercept. This can be:

a horizontal stretch if $b \neq -1$

a Horiz. reflection into the y-axis if $b = -1$

so yes, more than one option exists.