p 038 q1

Practise

1. The function $y = x^2$ has been transformed to $y = af(bx)$. Determine the equation of each transformed function.

a) Its graph is stretched horizontally about the y-axis by a factor of 2 and then reflected in the x-axis.

b) Its graph is stretched horizontally about the y-axis by a factor of $\frac{1}{4}$, reflected in the y-axis, and then stretched vertically about the x-axis by a factor of $\frac{1}{4}$.

a) using $g(x) = a[f(b(x-h))] + k$ ← general form

but since we are using a simpler model, it's only

$$g(x) = af(bx)$$

$g(x) = af(bx)$

- a: vert stretch and/or V. reflection
- b: Horiz. stretch by a factor of $\frac{1}{b}$

The meaning of the parameters a & b

1) H. stretch by a factor of 2 $\Rightarrow$ $b = \frac{1}{2}$

2) reflected into the x-axis means a vertical stretch by a factor of -1

2) reflected into the x-axis means a vertical stretch by a factor of -1

i.e. $a = -1$

Compounding these two we get

$$\boxed{g(x) = -f\left(\frac{1}{2}x\right)}$$

Since we know that $f(x) = x^2 \Rightarrow f\left(\frac{1}{2}x\right) = \left(\frac{1}{2}x\right)^2$

we can calculate further:

$$g(x) = -\left[\left(\frac{1}{2}x\right)^2\right] = -\left(\frac{1}{2^2}\cdot x^2\right)$$

$$\boxed{g(x) = -\frac{1}{4}x^2}$$

b) $g(x) = af(bx)$

- a: Vert stretch and/or V. reflection
- b: Horiz. stretch by a factor of $\frac{1}{b}$ (can also be a horiz. reflection in the y-axis if $b = -1$)

$a = \frac{1}{4}$

$\frac{1}{b} \downarrow$

$\frac{1}{4} = \frac{1}{b}$

$\Downarrow$

$b_2 = 4$

overall $b = b_1 \times b_2 = (-1)(4)$

overall $b = b_1 \times b_2 = (-1)(4)$

$b = -4$

$g(x) = \frac{1}{4} f(4x)$

$f(x) = x^2$

$\Rightarrow g(x) = \frac{1}{4}(-4x)^2$

$g(x) = \frac{1}{4} \cdot (-4)^2 x^2$

$g(x) = \frac{1}{4} \cdot \overset{4}{\cancel{16}} \cdot x^2$

$\boxed{g(x) = 4x^2}$