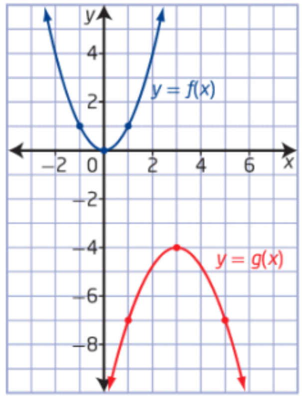
Your Turn

The graph of the function $y = g(x)$ represents a transformation of the graph of $y = f(x)$. State the equation of the transformed function. Explain your answer.



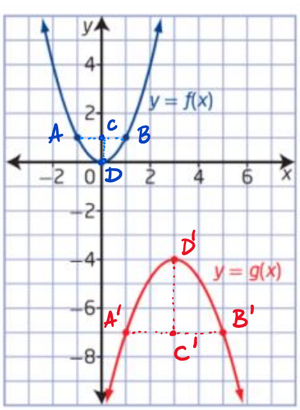
There are probably several ways to solve this. But some simple moves can take us from $f(x)$ to $g(x)$.

The blue function is a parabola w/ one root at $x=\phi$ & we can read from the graph that

$$f(-2)=f(2)=4=2^2$$
$$f(-1)=f(1)=1=1^2 \quad *$$

So its safe to assume $\boxed{f(x)=x^2}$ we can use this later.

The diagram also shows a set of points that seem to indicate a transitson from blue points to red points, but we can see that the distance between them has been enlarged from a width of 2 units to 4 units between AB and A'B'



indicating a horizontal stretch of some sort.

Since CB represents the distance from zero, we can use it to say x_B = distance C to B

$CB = 1$ unit

& the change is

$C'B' = 2$ units, so it

has double.

Probably $(x,y) \to (2x, y)$ here

and we can verify this later

the horizontal stretch factor = 2

** $\Rightarrow$ the function definition will use $\frac{1}{2}$.

Also the distance between CD & C'D'

has been increased from CD = 1 unit

to C'D' = 3 unit, indicating some sort of

a vertical stretch.

V. distance: $1 \to 3 = 3 \times 1$

Probably this vertical stretch factor is 3:

$(x, y) \to (x, 3y)$ ***

we'll verify this @ the end.

Additionally to the stretches it seems that a few other moves are necessary. We can trade these by examining what happens to the vertex of the blue parabola:

A horizontal shift to the Right will take the vertex to $x = 3$, and flip it upside down and then shift downwards by 4 units, u get the form of $y = g(x)$. Let's list all in order, then try to build the mapping from which we can build the function definition of $g(x)$.

Step 1: A horizontal stretch by a factor of 2 and a vertical stretch by a factor of 3, these two in any order since the two dimensions: vertical (y) and horizontal (x) do not interfere with each other.

from ** and *** $(x,y) \to (2x, 3y)$

$$g_1(x) = 3f(\tfrac{1}{2}x)$$

Step 2: Shift Right by 3 units to arrive @ $x=3$ from $x=0$: $(x,y) \to$ on step 1 $(x+3, y)$

$$g_2(x) = g_1(x-3)$$

$$g_2(x) = 3f(\tfrac{1}{2}(x-3))$$

H. stretch goes 1st.

Step 3: Flip:
Vertical reflection into the x-axis
$(x,y) \to (x,-y)$ on step 2

$$g_3(x) = -g_2(x)$$

$$g_3(x) = -3f(\tfrac{1}{2}(x-3))$$

$$g_3(x) = -3f\left(\tfrac{1}{2}(x-3)\right)$$

Step 4: Shift down by 4 units

$$(x,y) \xrightarrow{\text{on step 3}} (x, y-4)$$

$$g_4(x) = g_3(x) - 4$$

$$g_4(x) = -3f\left(\tfrac{1}{2}(x-3)\right) - 4$$

this is our final transformation $g(x)$.

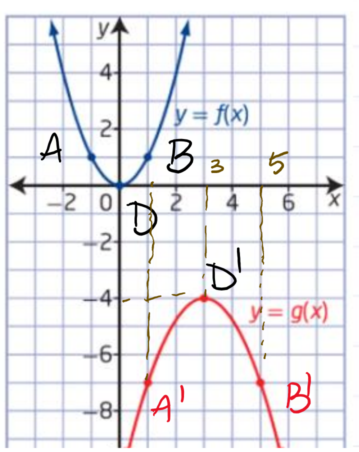
$$\therefore \boxed{g(x) = -3f\left(\tfrac{1}{2}(x-3)\right) - 4}$$

we can leave this so or use $f(x) = x^2$ to get a formula that does not involve $f(x)$:

$$g(x) = -3\left[\tfrac{1}{2}(x-3)\right]^2 - 4 = -3\left(\tfrac{1}{4}(x-3)^2\right) - 4 =$$

$$\boxed{g(x) = -\frac{3}{4}(x-3)^2 - 4}$$

Let's verify:



$f(x) = x^2$

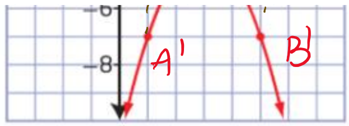
Let's try points:

$A, B, D \longrightarrow A', B', D'$

based on $g(x) = -3f\left(\tfrac{1}{2}(x-3)\right) - 4$

or

$g(x) = -\frac{3}{4}(x-3)^2 - 4$



$$g(x) = -\frac{3}{4}(x-3)^2 - 4$$

First read the points from the red graph:

$A'(1,-7)$

$B'(5,-7)$

$D'(3,-4)$

$$g(1) = -\frac{3}{4}(1-3)^2 - 4 = -\frac{3}{4}(-2)^2 - 4 = -\frac{3}{\cancel{4}} \cdot \cancel{4} - 4 = -3-4 = -7 \leftarrow$$ correct for point A'

$$g(5) = -\frac{3}{4}(5-3)^2 - 4 = -\frac{3}{4} \cdot (2^2) - 4 = -\frac{3}{\cancel{4}} \cdot \cancel{4} - 4 = -3-4 = -7 \leftarrow$$ correct for point B'

$$g(3) = -\frac{3}{4}(3-3)^2 - 4 = -\frac{3}{4}(0^2) - 4 = -\frac{3}{4} \cdot 0 - 4 = 0-4 = -4$$

$\uparrow$ correct for point D.

Therefore we assume the formula calculated @ is correct.

See also the solution in Desmos:

https://www.desmos.com/calculator/g2lmmuwbim