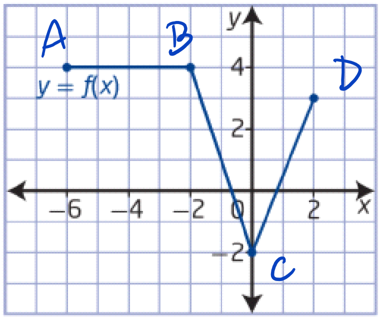
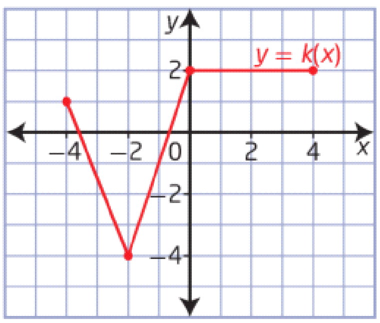
p 039 q4

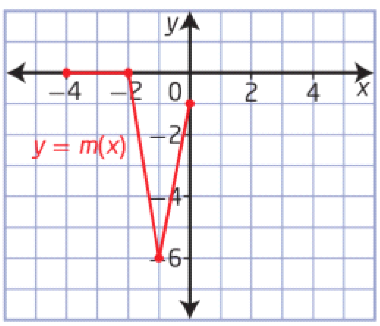
4. Using the graph of $y = f(x)$, write the equation of each transformed graph in the form $y = af(b(x - h)) + k$.



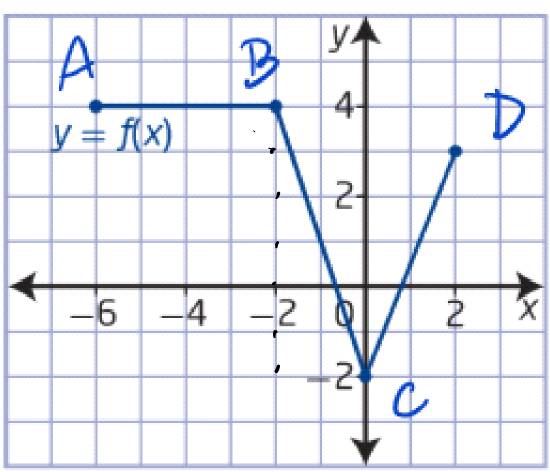
a)

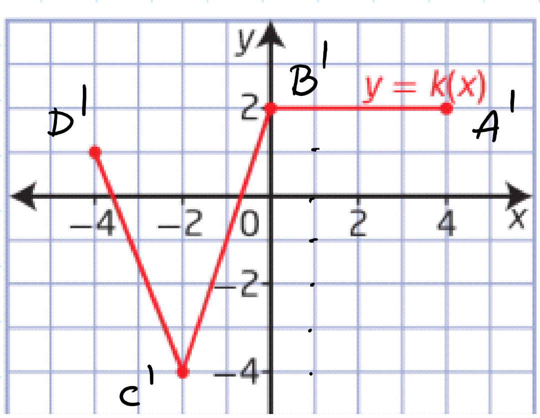


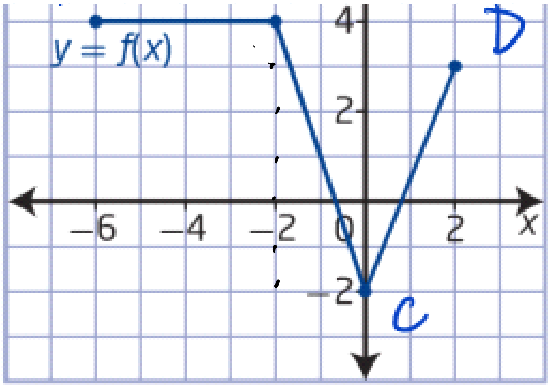
b)

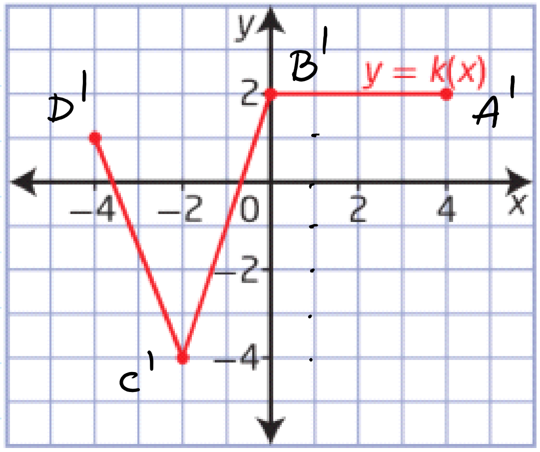


a) We can guess which point is which.









It doesn't look like a stretch happened vertically

or horizontally: $AB = A'B' = 4$ units.

the height difference between B & C
is 6 unit (height! not distance.)

etc.

Thus we can say. $\boxed{a=1}$ & $b = 1$ b_1

Step 1) We can see that $C \to C'$ is a 2 unit descent
$\boxed{k=-2}$

Step 2) We can see a Horizontal flip => a Horiz. Reflection
into the y-axis
$\Rightarrow b_2 = -1$

$b_1 = 1$

$b = b_1 \times b_2 = 1 \times (-1) = -1$
$\boxed{b=-1}$

Step 3) We can see that originally B was
2 units away from the y-axis

Step 3) We can see that originally B was 2 units away from the y-axis

option 1

it flipped then moved Left by 2 units

option 2

it moved Left 2 units to then it flipped.

both orders gives us the same thing

a shift Left by 2 units

$$\Downarrow$$

$$\boxed{h = -2}$$

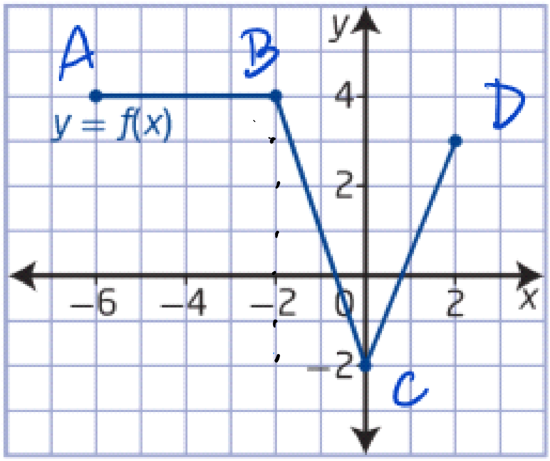
Now we can put them all together:

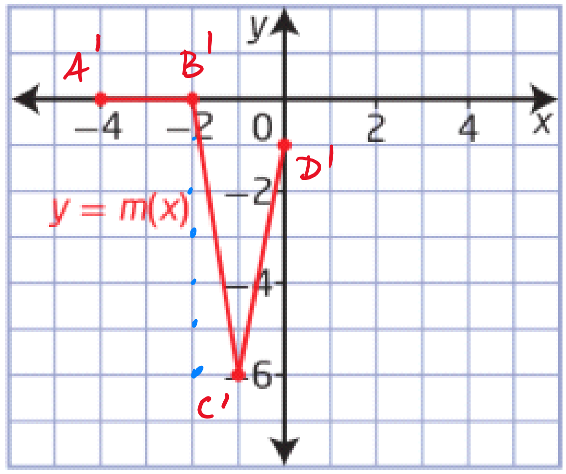
$$g(x) = \underbrace{a}_{1} f(\underbrace{b}_{-1}(x - \underbrace{h}_{-2})) + \underbrace{k}_{-2}$$

$$g(x) = f(-(x-(-2)) - 2$$

$$\boxed{g(x) = f(-(x+2)) - 2}$$

b) Again, we can guess the position of the points





It is clear that horizontal distances have changed

$AB = 4$, $A'B' = 2$ } => a Horiz. Stretch happened. by half

$x \to \frac{1}{2}x \Rightarrow b = \frac{1}{\frac{1}{2}} = 2$

$\boxed{b=2}$

we can also verify the H. stretch factor on the horiz. distance btwn B & D for ex:

x_B to x_D is 4 units => $x_{B'}$ to $x_{D'}$ should be half that: 2 units.

indeed the red graph shows that is the case.

So $b=2$ is correct.

the height btwn B and C is 6
B' C' 6 } ← so no vertical stretch happened

$\Downarrow$

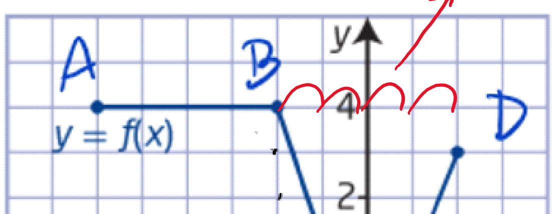
$\boxed{a=1}$

We can also see that the red graph was not reflected into the x or y axes.

But the Horiz. stretch was probably followed by a Horiz. Shift, probably to the Left.

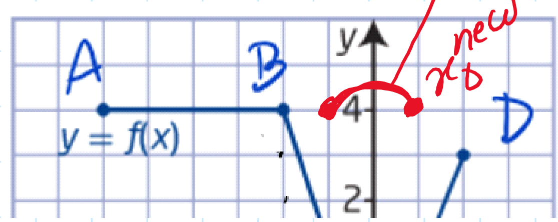
Let's check: when the horizontal distance $x_B x_D$ contracts from 4 to 2, the points B & D get closer to the y-axis as shown

4 units apart



2 units apart

new x_D



B & D symmetric in relation to the y-axis

$x_D = 2$

also symmetric after the H. stretch.

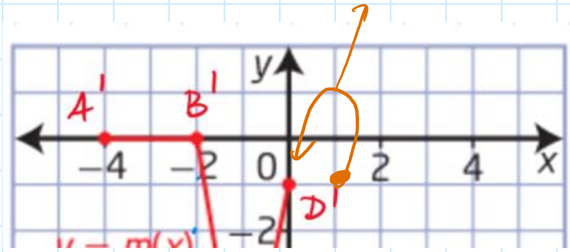
$x_D^{new} = 1$

Thus point D would have moved 1 unit to the Left b/c $x_D' = \emptyset$

Thus a H. shift to the Left by 1 unit happened AFTER the stretch.

$\Downarrow$

$\boxed{h = -1}$



There is one more thing: a shift down of point C

$y_C = -2 \quad \rightarrow \quad y_{C'} = -6$

the shift is by 4 units down $\Rightarrow$ $\boxed{k = -4}$

the shifft is by 4 units down $\Rightarrow$ $\boxed{k = -4}$

When we pull them all together we get

$a = 1,\ b = 2,\ h = -1$ & $k = -4$

$\Rightarrow g(x) = f(2(x-(-1)) - 4$

$$\boxed{g(x) = f(2(x+1)) - 4}$$