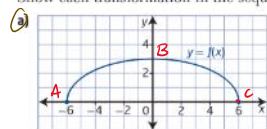
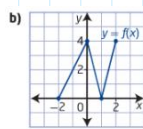


5. For each graph of  $y = f(x)$ , sketch the graph of the combined transformations. Show each transformation in the sequence.



- Step 1 • vertical stretch about the x-axis by a factor of 2
- Step 2 • horizontal stretch about the y-axis by a factor of  $\frac{1}{3}$
- Step 3 • translation of 5 units to the left and 3 units up



- vertical stretch about the x-axis by a factor of  $\frac{3}{4}$
- horizontal stretch about the y-axis by a factor of 3
- translation of 3 units to the right and 4 units down

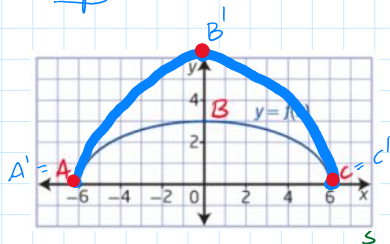
a) we can start by listing a few points & apply the list of transformations in order while we describe the transf. as a mapping of either the  $x$  or the  $y$  coordinate:

|   | $(x, y)$  | Step 1<br>$y \rightarrow 2y$ | Step 2<br>$x \rightarrow \frac{1}{3}x$ | Step 3<br>$x \rightarrow x - 5$<br>$y \rightarrow y + 3$ |
|---|-----------|------------------------------|----------------------------------------|----------------------------------------------------------|
| A | $(-6, 0)$ | $(-6, 0)$                    | $(-\frac{6}{3}, 0) = (-2, 0)$          | $(-2 - 5, 0 + 3) = (-7, 3) = A'$                         |
| B | $(0, 3)$  | $(0, 6)$                     | $(\frac{0}{3}, 6) = (0, 6)$            | $(0 - 5, 6 + 3) = (-5, 9) = B'$                          |
| C | $(6, 0)$  | $(6, 0)$                     | $(\frac{6}{3}, 0) = (2, 0)$            | $(2 - 5, 0 + 3) = (-3, 3) = C'$                          |

We are required to show each transformation.

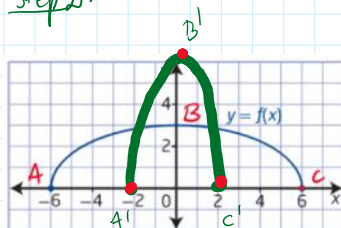
We can start by marking the points in each step & then just connecting them to obtain the graph lines.

Step 1



$$g_1(x) = 2f(x)$$

Step 2:



$$g_2(x) = g_1(3x)$$

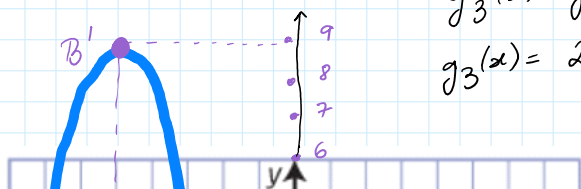
$$g_2(x) = 2f(3x)$$

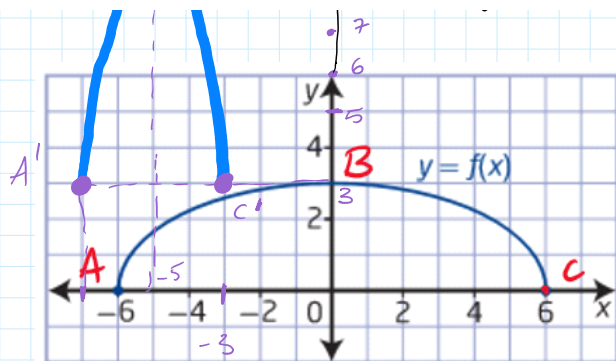
Step 3

bc the H. mapping is -5

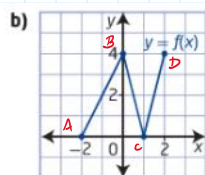
$$g_3(x) = g_2(x + 5) + 3$$

$$g_3(x) = 2f(3(x - 5)) + 5$$





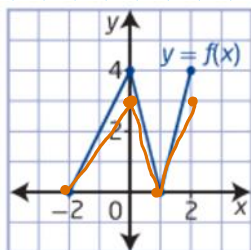
b)



We'll take a similar approach, mark points & then extend the lines.

- Step 1
- vertical stretch about the x-axis by a factor of  $\frac{3}{4}$
- Step 2
- horizontal stretch about the y-axis by a factor of 3
- Step 3
- translation of 3 units to the right and 4 units down

Step 1:  $(x, y) \rightarrow (x, \frac{3}{4}y)$



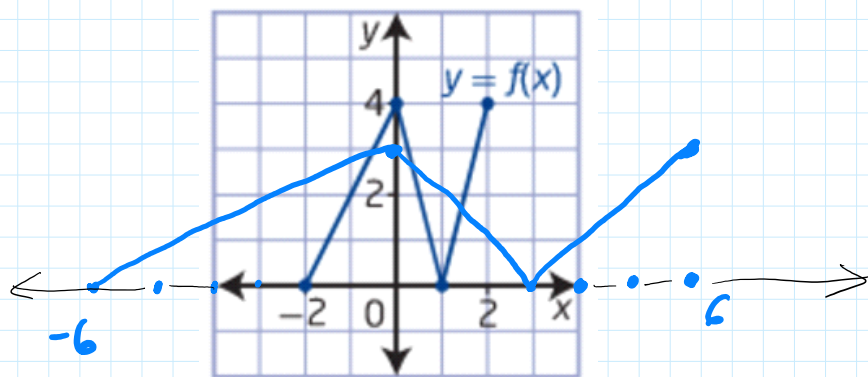
|   | $(x, y)$  | Step 1<br>$y \rightarrow \frac{3}{4}y$ |
|---|-----------|----------------------------------------|
| A | $(-2, 0)$ | $(-2, 0)$                              |
| B | $(0, 4)$  | $(0, \frac{3}{4} \cdot 4) = (0, 3)$    |
| C | $(1, 0)$  | $(1, 0)$                               |
| D | $(2, 4)$  | $(2, \frac{3}{4} \cdot 4) = (2, 3)$    |

$$g_1(x) = \frac{3}{4}f(x)$$

Step 2  $(x, y) \rightarrow (3x, y)$

$$g_2(x) = \frac{3}{4}f(3x)$$

|   | $(x, y)$  | Step 1<br>$y \rightarrow \frac{3}{4}y$ | Step 2<br>$x \rightarrow 3x$ |
|---|-----------|----------------------------------------|------------------------------|
| A | $(-2, 0)$ | $(-2, 0)$                              | $(-6, 0)$                    |
| B | $(0, 4)$  | $(0, \frac{3}{4} \cdot 4) = (0, 3)$    | $(0, 3)$                     |
| C | $(1, 0)$  | $(1, 0)$                               | $(3, 0)$                     |
| D | $(2, 4)$  | $(2, \frac{3}{4} \cdot 4) = (2, 3)$    | $(6, 3)$                     |



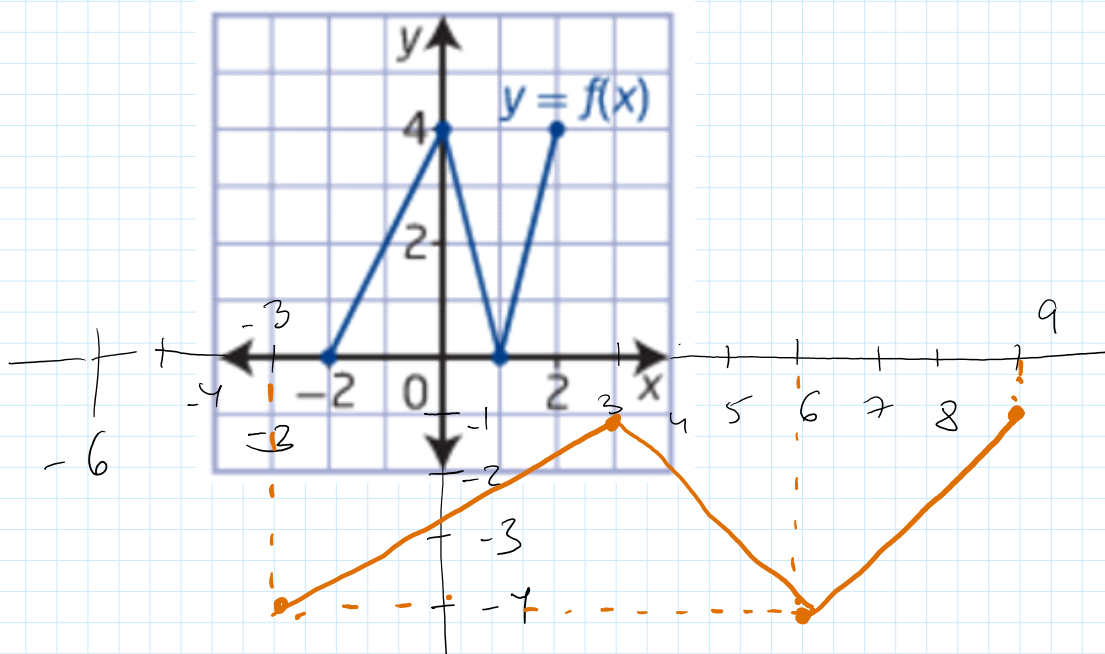
Step 3  $(x, y) \rightarrow (x+3, y-4)$

$$g_3(x) = g_2(x-3) - 4$$

$$\leftarrow g_2(x) = \frac{3}{4} f(3x)$$

$$g_3(x) = \frac{3}{4} \cdot f(3(x-3)) - 4$$

|   | $(x, y)$  | Step 1<br>$y \rightarrow \frac{3}{4}y$ | Step 2<br>$x \rightarrow 3x$ | Step 3.<br>$(x, y) \rightarrow (x+3, y-4)$ |
|---|-----------|----------------------------------------|------------------------------|--------------------------------------------|
| A | $(-2, 0)$ | $(-2, 0)$                              | $(-6, 0)$                    | $(-6+3, 0-4) = (-3, -4)$                   |
| B | $(0, 4)$  | $(0, \frac{3}{4} \cdot 4) = (0, 3)$    | $(0, 3)$                     | $(0+3, 3-4) = (3, -1)$                     |
| C | $(1, 0)$  | $(1, 0)$                               | $(3, 0)$                     | $(3+3, 0-4) = (6, -4)$                     |
| D | $(2, 4)$  | $(2, \frac{3}{4} \cdot 4) = (2, 3)$    | $(6, 3)$                     | $(6+3, 3-4) = (9, -1)$                     |



- 6

