

6. The key point $(-12, 18)$ is on the graph of $y = f(x)$. What is its image point under each transformation of the graph of $f(x)$?

a) $y + 6 = f(x - 4)$

b) $y = 4f(3x)$

c) $y = -2f(x - 6) + 4$

d) $y = -2f\left(-\frac{2}{3}x - 6\right) + 4$

e) $y + 3 = -\frac{1}{3}f(2(x + 6))$

a) $y + 6 = f(x - 4)$

$$y = f(x - 4) - 6$$

the mapping is: $(x, y) \rightarrow (x + 4, y - 6)$

$$\Rightarrow (-12, 18) \rightarrow (-12 + 4, 18 - 6) = (-8, 12)$$

The img. point is $(-8, 12)$.

b) $y = 4f(3x)$

the mapping is: $(x, y) \rightarrow \left(\frac{1}{3}x, 4y\right)$

$$\Rightarrow (-12, 18) \rightarrow \left(\frac{1}{3}(-12), 4 \cdot 18\right) = (-4, 72)$$

the img pnt is: $(-4, 72)$.

c) $y = -2f(x - 6) + 4$

the mapping is $(x, y) \rightarrow (x + 6, -2y + 4)$

the mapping is $(x, y) \rightarrow (x+6, -2y+4)$

$$\Rightarrow (-12, 18) \rightarrow (\underbrace{-12+6}_{-6}, \underbrace{-2(18)+4}_{-36+4}) = (-6, -32)$$

$$d) y = -2f\left(-\frac{2}{3}x - 6\right) + 4$$

before we deal w/ the mapping, we should

format this in the traditional: stretch first

shift after:

$$\text{Horizontal parameters} \left\{ \left(-\frac{2}{3}x - 6 \right) = -\frac{2}{3} \left(\frac{-2}{3}x - \frac{6}{-\frac{2}{3}} \right) = \right.$$

we want to factor the entire stretch!

$$= -\frac{2}{3} \left(x - 6 \cdot \left(-\frac{3}{2} \right) \right) =$$

$$= -\frac{2}{3} (x + 9)$$

$$\Rightarrow y = -2f\left(-\frac{2}{3}(x+9)\right) + 4$$

$\Rightarrow$ the mapping (overall, not just horizontal) is:

$$(x, y) \rightarrow \left(\frac{1}{-\frac{2}{3}}x - 9, -2y + 4 \right)$$

$$(x, y) \rightarrow (-\frac{3}{2}x - 9, -2y + 4)$$

$$(x, y) \rightarrow \left(-\frac{3}{2}x - 9, -2y + 4 \right)$$

$$\Rightarrow (-12, 18) \rightarrow \left(-\frac{3}{2}(-12) - 9, -2 \cdot 18 + 4 \right)$$

$$= (+18 - 9, -36 + 4)$$

$$= (9, -32)$$

the img. pt is $(9, -32)$.

$$e) \quad y + 3 = -\frac{1}{3}f(2(x+6))$$

$$y = -\frac{1}{3}f(\underbrace{2(x+6)}) - 3$$

formatted well, w/ brackets
to indicate stretch first
& shift after

the mapping is:

$$(x, y) \rightarrow \left(\frac{1}{2}x - 6, -\frac{1}{3}y - 3 \right)$$

$$\Rightarrow (-12, 18) \rightarrow \left(\frac{1}{2}(-12) - 6, -\frac{1}{3} \cdot 18 - 3 \right) =$$

$$= (-6 - 6, -6 - 3)$$

$$= (-12, -9)$$

the image point is $(-12, -9)$.