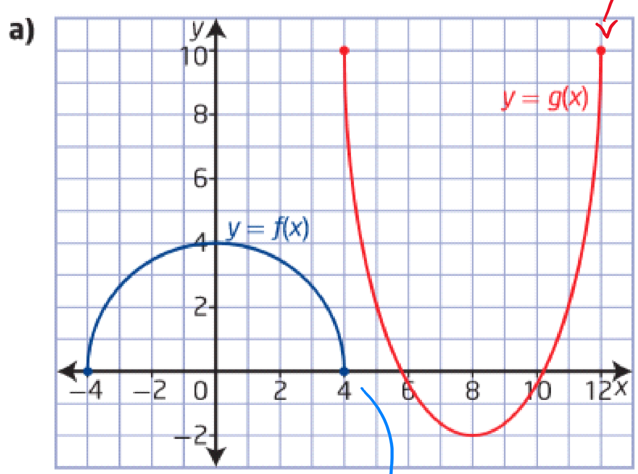
p 040 q10 (a)

10. The graph of the function $y = g(x)$ represents a transformation of the graph of $y = f(x)$. Determine the equation of $g(x)$ in the form $y = af(b(x - h)) + k$.

notice these dots in red

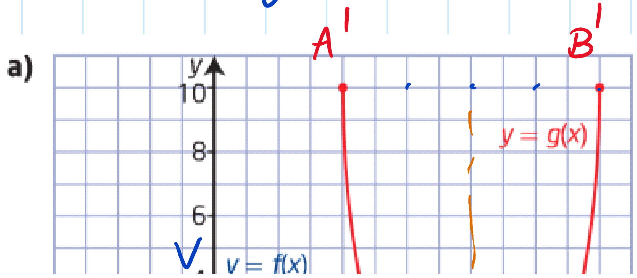


Let's assume $f(x)$ is a parabola (even though it looks more like a semi-circle)

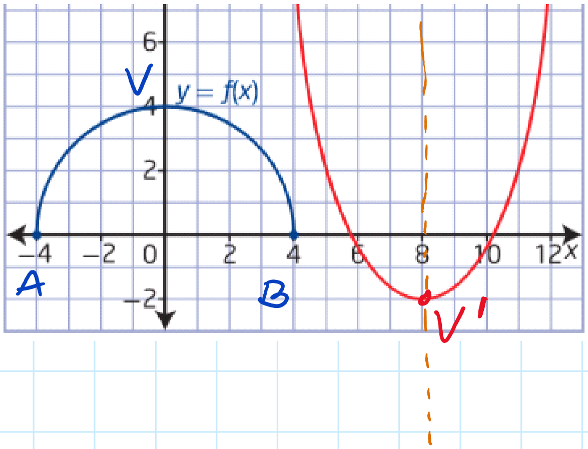
see there are dots in blue as well

We will assume the dots in blue have transitioned to the dots in red, lets call them A & B in $f(x)$ and A' & B' in $g(x)$.

We could also mark a vertex V & V' on each since every parabola has one, and resides on the parabola's axis of symmetry. On $f(x)$ that is $x = \phi$ & on $g(x)$ the axis of symmetry is $x = 8$, which we can see on the graph.



The function $g(x)$ looks stretched & inverted, i.e reflected vertically **



& inverted, i.e reflected vertically into the x-axis. (**)

What kind of stretch?

- vertical for sure b/c the distance between V & A and B has increased
- horizontal distance between A B remains 8 units when A'B' is counted. (*)

There is also a vertical shift which moved V to a different position after the vertical reflection. (***) And clearly a H. shift as well potentially to the Right.

Let's use the traditional orders: Stretch first w/ Horizontals first, then shifts, so the Horizontals go first as well.

Attempt

1) H. stretch → we said none likely earlier (see (*))
$\Rightarrow \boxed{b=1}$

2) V. stretch
the dist from point V to line AB = 4 units.
the dist " " V' " " A'B' = 12 units

counted on
the red graph.

this could b a stretch by a
factor of 3 b/c
$4 \rightarrow 3 \times 4 = 12$

$\Rightarrow a = 3$

3) See ** : $\Rightarrow$ a needs to be negative.

$\Rightarrow \boxed{a = -3}$

4) see *** where would a vertical
reflection accompanied by the stretch
move the vertex?

$V(0,4) \longrightarrow V'(8,-2)$

we read this
from the
red graph
based on $x = 8$
which appears to be
the axis of symmetry, and
the vertex must reside on top
of it.

For now we just care about the y:

y: $4 \rightarrow -2$

$y: \quad 4 \to -2$

$y \to -3y$ (from part 3))

Let's verify that this is enough to take 4 to -2

$\underset{f}{y} = 4 \Rightarrow \quad \underset{g}{y} = -3(4) = \underbrace{-12}_{\text{thus not } -2}$

you would have to move up to -2 as follows using a vertical shift:

$-12 + k = -2$ ← the actual position of V'

$\Downarrow$

$k = -2 + 12$

$k = 10$

Thus a move up by 10 is necessary after the V. refl. and V. stretch.

$\Rightarrow \boxed{k = 10}$

$V(0,4) \to V'(8,-2)$ can also help us w/ the Horiz. shift, which does not interfere w/ the vertical shift

x only: $0 \longrightarrow 8$

we'll assume $x \rightarrow x+8$

this is a shift to the Right by 8 units.

$\therefore \boxed{h=8}$

Let's glue it all together: $h=8$

$k=10$

$a=-3$

$b=1$

$\Rightarrow \boxed{g(x) = -3f(x-8)+10}$