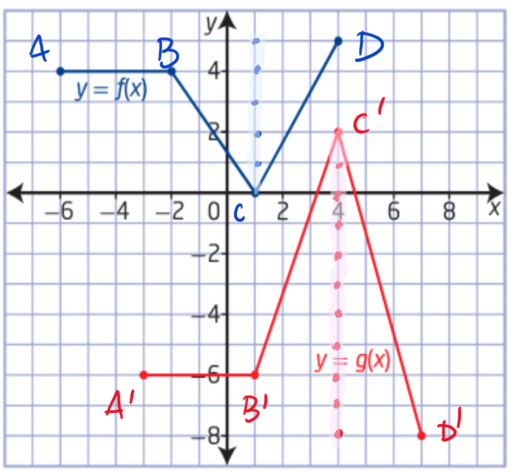
p 040 q10 (b)

10. The graph of the function $y = g(x)$ represents a transformation of the graph of $y = f(x)$. Determine the equation of $g(x)$ in the form $y = af(b(x - h)) + k$.

b)



Some quick things that can b identified

(i) – V. refl. into x-axis

(ii) – V. stretch

(iii) – H. distances have been maintained (count them on the graphs)

(iv) – V. translation, likely downwards

If we use the traditional order, can fit the form $y = af(b(x-h)) + k$

1) We start w/ b, nothing happens – see (iii)

$\boxed{b = 1}$

2) a = ? examine vertical distances:

y_C to y_D: 5 units.

$y_{C'}$ to $y_{D'}$: 10 units

$\Rightarrow$ double the vertical distance

$\Downarrow$

$5 \to 2 \times 5 = 10$

$\Downarrow$

$a = 2$

but it must be that a is negative because we can see that C has turned upside down in relation to points B & D. It was first below them but C' is above B' & D'.

$\Rightarrow$ a vertical reflection into the x-axis.

$$\Rightarrow \boxed{a = -2}$$

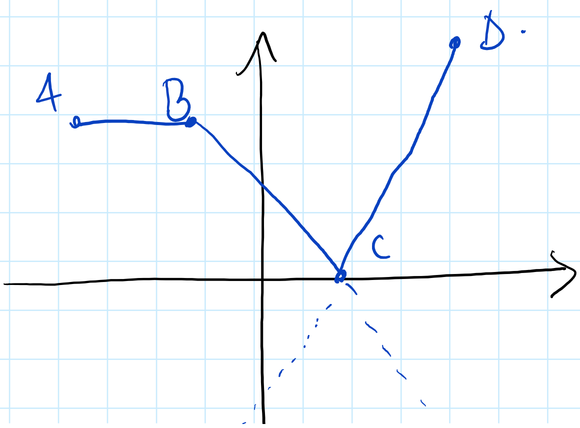
3) It looks like the graph moved 3 units to the Right since all points are 3 units away on the Right.

$$\Rightarrow \boxed{h = +3}$$

4) To figure out the V. shift, we must first know where the V. reflection and V. stretch has taken for ex. point $C_1(1,0)$ and then we can perform the shift up or down to the red graph at $C_1'(4,2)$

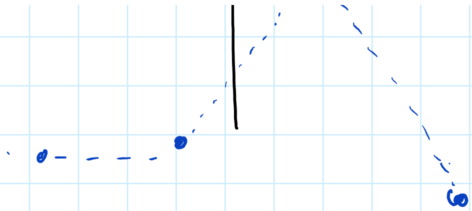
based on $a=-2 \Rightarrow (x,y) \to (\ldots, -2y)$

$$C(1,\underset{y}{0}) \to C'(\ldots, -2\cdot 0) = C'(.., 0)$$



this woul look roughly like so.

So clearly C would have to move up by <u>2 units</u>
We can then verify this is correct on the point $D(4,5) \to D'(7,-8)$



we can then
verify this is correct on the
point $D(4,5) \to D'(7,-8)$
(read the graphs)

So we think $\boxed{k=+2}$

It will help to verify this in the end...

The conclusion so far is that
- $a=-2$
- $b=1$
- $h=3$
- $k=2$

Lets use the mapping to check for example that point D moves correctly

$$(x,y) \to (x+3, -2y+2)$$

$$D(4,5) \to (4+3, -2\cdot 5+2) = (7, -10+2)$$
$$= (7,-8)$$

this is indeed point D'.

$$\therefore \boxed{g(x) = -2f(x-3)+2}$$ based on the coefficients calculated above.