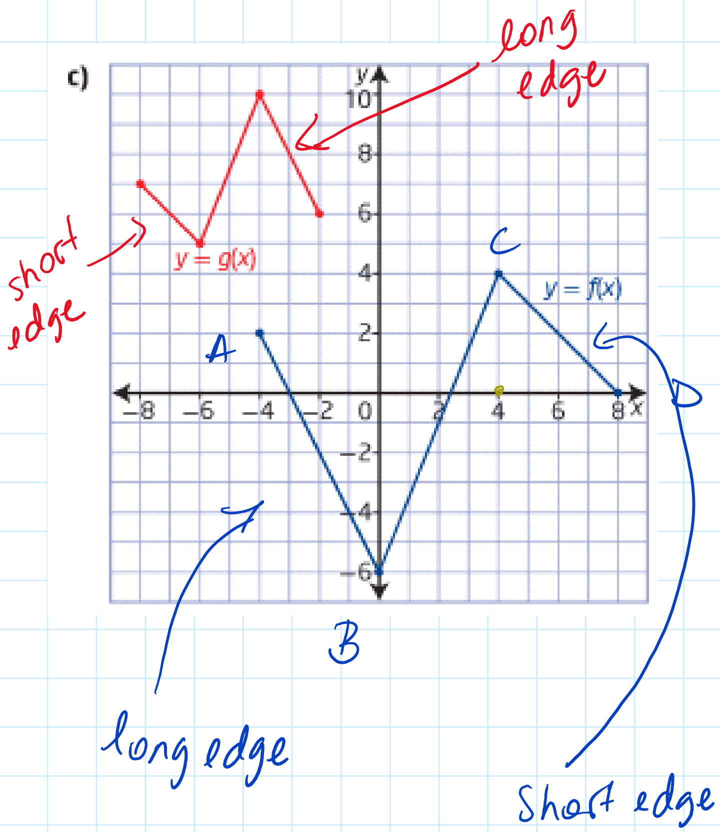
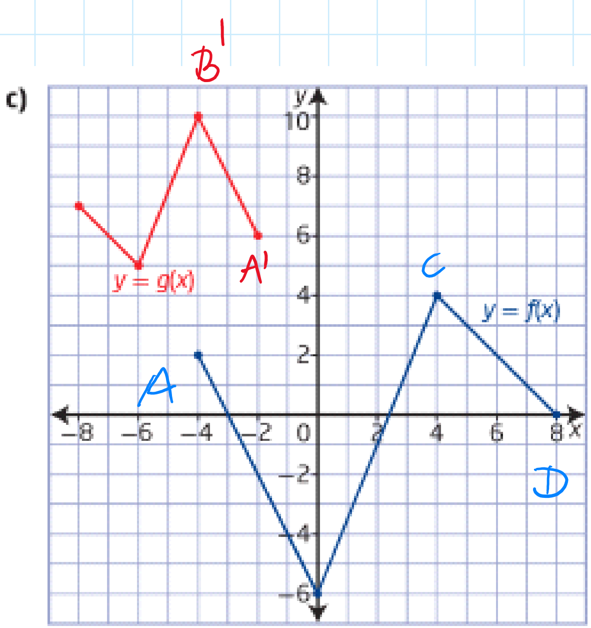
p 040 q10 (c)

10. The graph of the function $y = g(x)$ represents a transformation of the graph of $y = f(x)$. Determine the equation of $g(x)$ in the form $y = af(b(x - h)) + k$.



This is a bit trickey. First we must identify where each point has moved on the red graph.

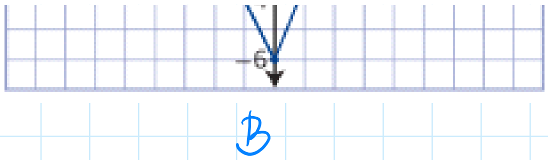
We must look which are the long edges & which are the short ones. This helps us identify A', B', C', and D'.



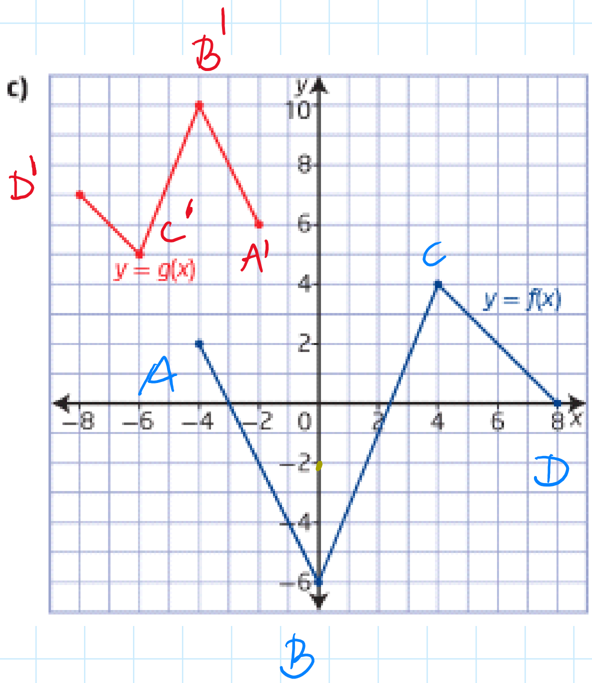
We focus on the long edge, the last point is A & A'

B' must not be an ending point.

From here it is easy to identify



From here it is easy to identify the remaining points: D' should be an ending point and C' must not.



Now with this arrangement we can say the red graph incurred both a vertical & horizontal stretch, a vertical reflection b/c B & C have flipped upside down between A & D, but also A & D have reflected horizontally into the y-axis since their positions is also flipped.

Again, we proceed w/ the traditional order:

b first
a next
h next
and then k

1) b ≡ H. stretch & reflection
clearly b must b negative

we count horizontal distances:

x_A to x_D : 12 units

$x_{A'}$ to $x_{D'}$: 6 units => that is half

this is the stretch factor

$-\frac{1}{2}$, but this is the mapping, we need the reciprocal in the function definition

$$b = \frac{1}{-\frac{1}{2}} = 1 \times \left(-\frac{2}{1}\right) = -2$$

$$\boxed{b = -2}$$

2) $a \equiv$ vertical stretch and reflection.

we count vertical distances:

y_B to y_C : 10 units

$y_{B'}$ to $y_{C'}$: 5 unit → this is half and it is the vert. stretch factor, and it must b negative to accomodate the reflection

$$\boxed{a = -\frac{1}{2}}$$

3) Let's see where some of the points went

so far, & then we can figure out the shifts:

$$\begin{array}{c} \color{red}{\tfrac{-1}{b}} \qquad \color{red}{a} \\ \uparrow \qquad\quad \uparrow \\ (x,y) \to \left(-\tfrac{1}{2}x,\ -\tfrac{1}{2}y\right) \end{array}$$ so far w/o the shifts

$$A(-4,2) \to \left(\tfrac{-1}{2}\overset{2}{(-\cancel{4})},\ \tfrac{-1}{\cancel{2}}\cdot\cancel{2}\right)$$

$$= (+2, -1)$$

but A' is located at $(-2,6)$

that would require additional

shifts: $(2,-1) \to (\underbrace{2-4}_{\text{to get } -2},\ \underbrace{-1+7}_{\text{to get } +6})$

H shift 4 units left

& V. shift 7 " up.

if this was correct, the overall mapping would be

$$(x,y) \to \left(-\tfrac{1}{2}x-4,\ -\tfrac{1}{2}y+7\right)$$

Let's test other points to verify this result

$$D(8,0) \to D'\left(\tfrac{-1}{\cancel{2}}\cdot\overset{4}{\cancel{8}}-4,\ -\tfrac{1}{2}\cdot 0+7\right)$$

$$= (-4-4,\ 0+7) = (-8,7)$$

$= (-4-4,\ 0+7) = (-8,7)$

this is indeed where D' resides so the mapping seems correct

one more point to verify:

$C(4,4) \to C'\left(-\frac{1}{\cancel{2}_1}(\cancel{4}^2)-4,\ \frac{-1}{\cancel{2}_1}(\cancel{4}^2)+7\right) =$

$= (-2-4,\ -2+7) = (-6,5)$

this is also indeed the point C' so the mapping is correct

$\therefore\ a = -\frac{1}{2},\ b = -2,\ h = -4,\ k = 7$

$g(x) = -\frac{1}{2} f\left(-2(x-(-4))\right) + 7$

$$\boxed{g(x) = -\frac{1}{2} f\left(-2(x+4)\right) + 7}$$