p 040 q11 (b)

11. Given the function $f(x)$, sketch the graph of the transformed function $g(x)$.

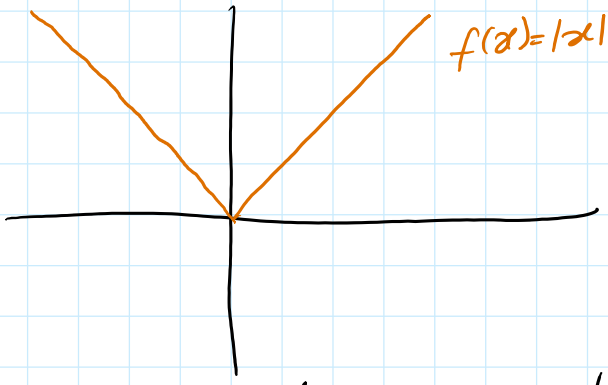
a) $f(x) = x^2$, $g(x) = -2f(4(x + 2)) - 2$

b) $f(x) = |x|$, $g(x) = -2f(-3x + 6) + 4$

c) $f(x) = x$, $g(x) = -\frac{1}{3}f(-2(x + 3)) - 2$

(b) we start by sketching the base parent function $f(x) = |x|$

This is a well known function that looks like so:



Now we can look at $g(x)$ and see it as series of steps that are transformations and will be easy to apply to the original graph one by one.

$$g(x) = -2f(-3x+6)+4$$

use traditional order of stretch before shift so you want to have -3 before brackets & x with no coefficient inside the brackets

like so: $f(-3x+6) = f\left(-3\left(\frac{-3x}{-3} + \frac{6}{-3}\right)\right)$

$= f(-3(x-2))$

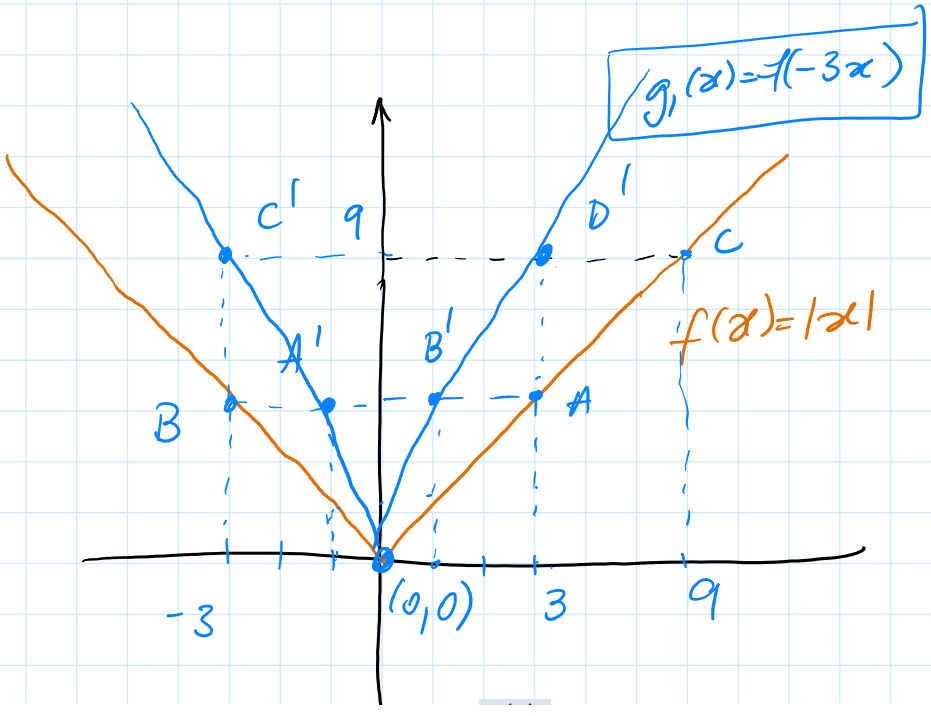
$\therefore \quad g(x) = \underbrace{-2}_{\text{second step}} f\underbrace{(-3(x-2))}_{\text{first step}} + \underbrace{4}_{\text{fourth step}}$ (third step)

First Step: $g_1(x)$

H. Reflect into the y-axis & stretch by a factor of $\frac{1}{3}$. Calculate a few points.

i.e. $(x, y) \to \left(-\frac{x}{3}, y\right)$

$A(3, 3) \to (-1, 3) = A'$
$B(-3, 3) \to (1, 3) = B'$
$C(9, 9) \to (-3, 9) = C'$
$D(-9, 9) \to (+3, 9) = D'$
$(0, 0) \to (0, 0)$



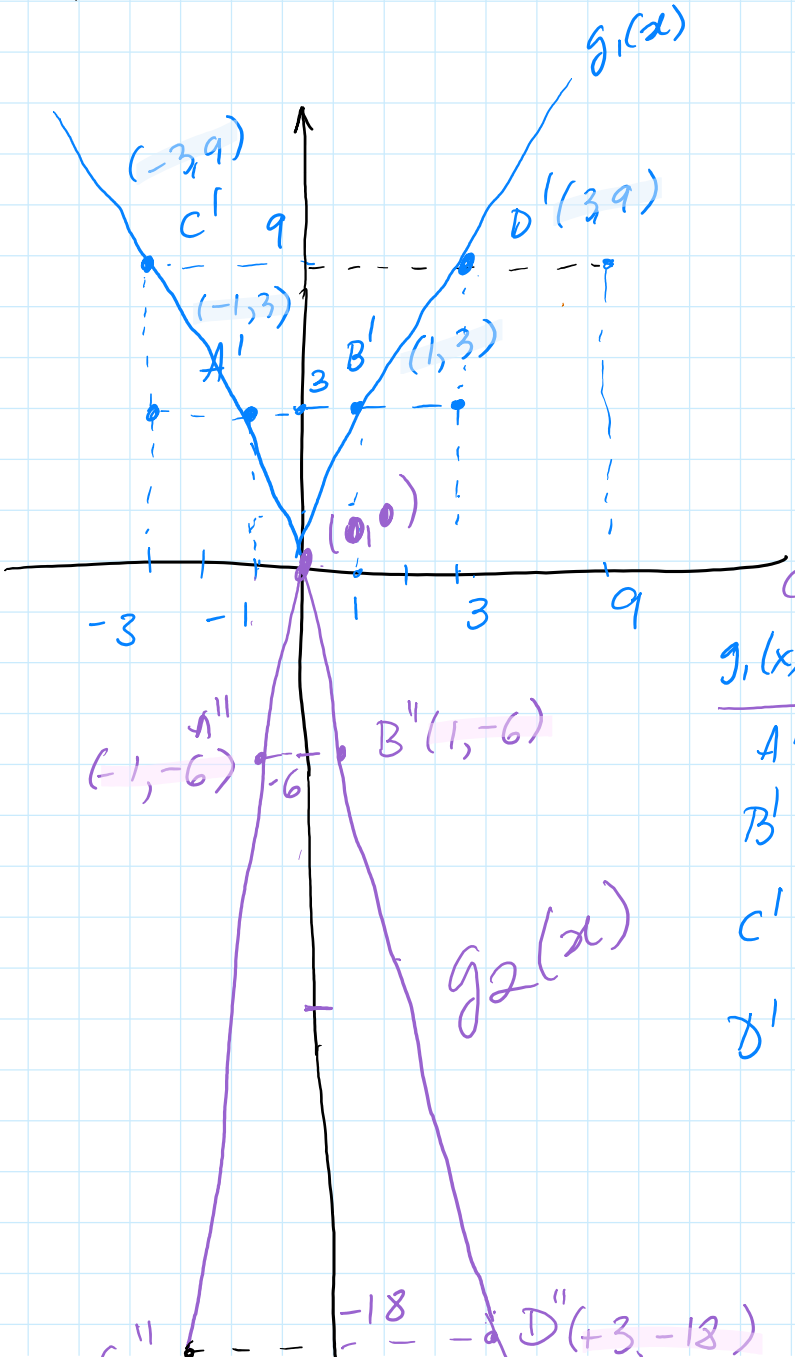
-3

g1(x):
https://www.desmos.com/calculator/cvk7yir0t2

Second Step is $g_2(x)$ applied to $g_1(x)$

$g_2(x) = -2g_1(x)$ ← the blue graph above

reflected & streched vertically by a factor of 2

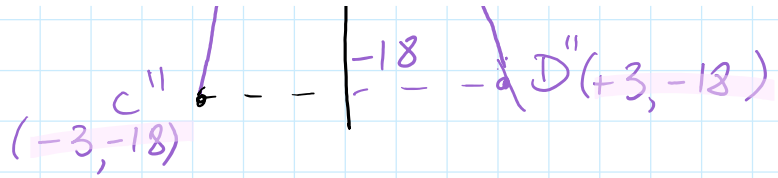


Calculate points:

$g_1(x)$	$(x,y) \to (x,-2y)$		$g_2(x)$
A'	(-1,3)	(-1,-6)	A''
B'	(1,3)	(1,-6)	B''
C'	(-3,9)	(-3,-18)	C''
D''	(3,9)	(3,-18)	D''
	(0,0)	(0,0)	

See here a nicer graph

g2(x):
https://www.desmos.com/calculator/17pnwnce8o



See here a nicer graph

g2(x):
https://www.desmos.com/calculator/17pnwnce8o

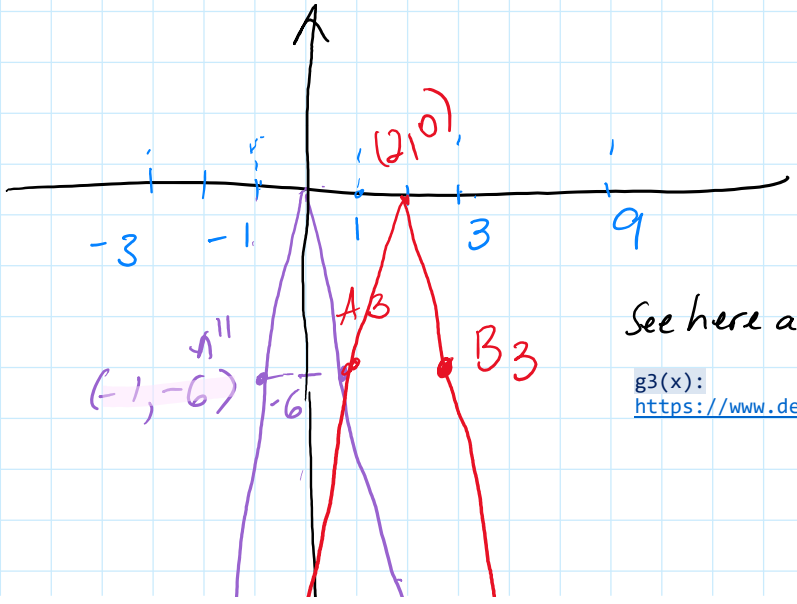
Third step is $g_3(x)$ applied to $g_2(x)$, i.e. the purple graph.

This is a Horizontal Shift 2 units to the Right.

$(x, y) \to (x+2, y)$

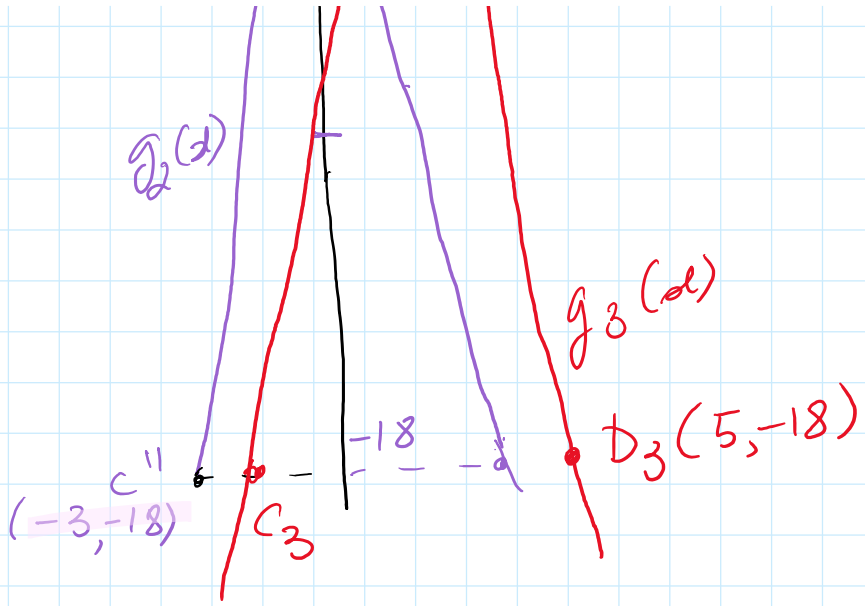
$g_3(x) = g_2(x-2)$

$g_2(x)$	(x, y)	$\to (x+2, y)$	$g_3(x)$
A''	$(-1, -6)$	$(1, -6)$	A_3
B''	$(1, -6)$	$(3, -6)$	B_3
C''	$(-3, -18)$	$(-1, -18)$	C_3
D''	$(3, -18)$	$(5, -18)$	D_3
	$(0, 0)$	$(2, 0)$	



See here a nicer graph:

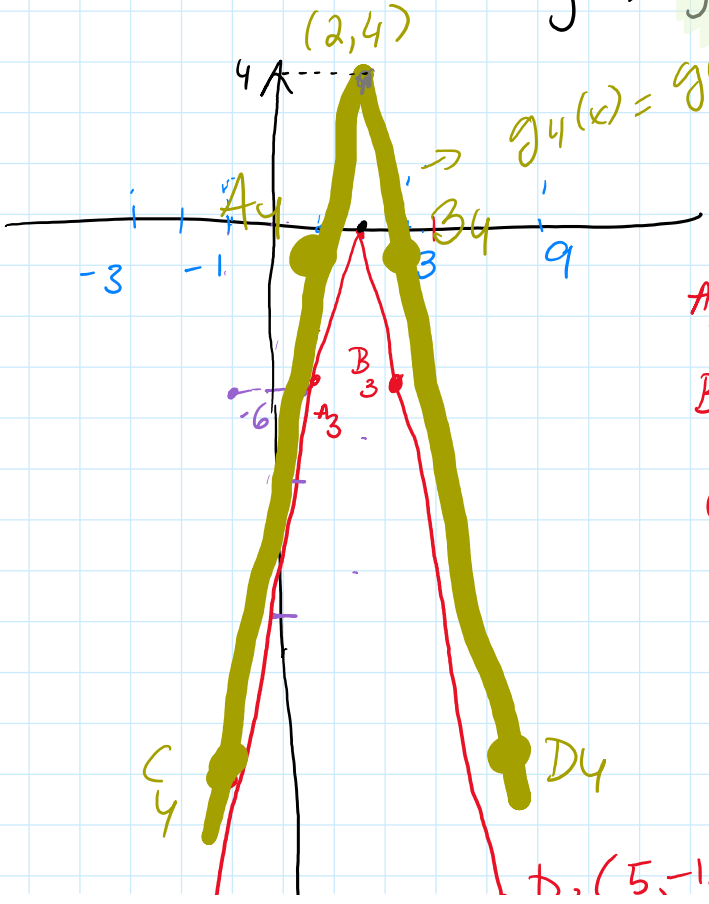
g3(x):
https://www.desmos.com/calculator/i6ozbcxm8h



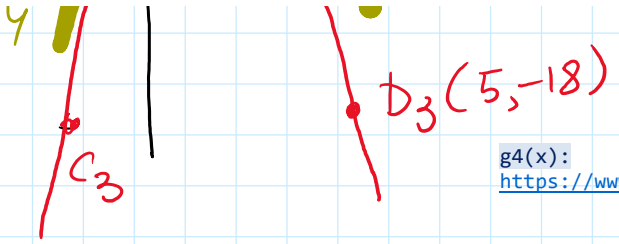
Fourth Step

A vertical shift of the red graph $g_3(x)$ up by 4 units. We get the final transformation

$$g(x) = g_4(x)$$



	$\to (x, y)$	$\to (x, y+4)$	
A_3	$(1, -6)$	$(1, -2)$	A_4
B_3	$(3, -6)$	$(3, -2)$	B_4
C_3	$(-1, -18)$	$(-1, -14)$	C_4
D_3	$(5, -18)$	$(5, -14)$	D_4
	$(2, 0)$	$(2, 4)$	



g4(x):
https://www.desmos.com/calculator/jbnkucvnni

Here is a more beautiful graph as plotted in Desmos.