p 040 q7

Apply

7. Describe, using an appropriate order, how to obtain the graph of each function from the graph of $y = f(x)$. Then, give the mapping for the transformation.

a) $y = 2f(x - 3) + 4$

b) $y = -f(3x) - 2$

c) $y = -\frac{1}{4}f(-(x + 2))$

d) $y - 3 = -f(4(x - 2))$

e) $y = -\frac{2}{3}f\left(-\frac{3}{4}x\right)$

f) $3y - 6 = f(-2x + 12)$

a) $y = 2f(x-3) + 4$ (*)

Vertical stretch by a factor of 2.

Horizontal Shift Right by 3 units

Vertical shift up by 4 units

(**)

The mapping: $(x, y) \rightarrow (x+3, 2y+4)$

remember that for for Horizontal Shifts the function definition (see (*)) and the function mapping (see (**)) have opposite signs.

The order:

1) A vertical stretch by a factor of 2.

2) Horizontal Shift Right by 3 units

3) A Vertical shift up by 4 units

This is the traditional order, but the following order would work just the

This is the traditional order, but the following order would work just the same; (2), (3) then (1).

Explanations: Traditionally we list all the stretches first, (and the Horizontal goes first) then the Horiz. Shift, then the Vertical Shift last.
However, we only need to worry not to interfere between the 2 dimensions, i.e. horizontal & vertical.
You can't mix the order of the Horizontal stretch w/ that of the Horizontal Shift, but you can mix a vertical stretch & a horizontal stretch to any order. You can also mix the order of a Horiz. shift w/ that of a Vertical shift.

It's safest to stay w/ the traditional order to avoid mistakes.

b) $y = -f(3x) - 2$

remember that a Horizontal stretch uses a reciprocal stretch factor in the mapping in comparison to the function definition.

mapping: $(x, y) \rightarrow \left(\frac{1}{3}x, -y-2\right)$

order: stretches first, shifts later and within those Horizontal goes traditionally first, even though traditionally first, even though Vertical would also work first.

↳ The following link shows an example $f(x)$, the transformation $g(x)$ as given here : $g(x) = -f(3x) - 2$ and a vertical order that goes first, to proove that it plots exactly the same transformation as the traditional order of $g(x)$.

https://www.desmos.com/calculator/pstd7vyllx

The order is therefore:

1) a Horiz. stretch by a factor of $\frac{1}{3}$

2) a Vertical Reflection into the y-axis (→ which is equivalent to a Vert. Stretch by a factor of -1).

3) a Vertical Shift down (b/s of the minus) by 2 units.

c) $y = -\frac{1}{4} f(-(x+2))$

mapping: $(x, y) \rightarrow \left(-x-2, -\frac{1}{4}y\right)$

order:

1) a Horizontal Refletion into the y axis (designated by the -1 in

1) a Horizontal Reflection into the y axis (designated by the -1 in front of the brackets)

2) a Vertical Reflection into the x-axis

3) a Vertical stretch by a factor of $\frac{1}{4}$.

4) a Horizontal Shift to the Left by 2 units.

d) $y-3=-f(4(x-2))$

$y=-f(4(x-2))+3$

$(x,y)\rightarrow\left(\frac{1}{4}x+2,\ -y+3\right)$

1) a H. stretch by a factor of $\frac{1}{4}$

2) a V. reflection into the x-axis

3) a H. shift 2 units to the Right

4) a V. shift 3 units up.

e) $y=-\frac{2}{3}f\left(-\frac{3}{4}x\right)$

V. reflection into the x-axis

V. stretch

H. reflection into the y axis → FIRST

into the x-axis → THIRD

V. stretch by a factor of $\frac{2}{3}$ → FOURTH

the y-axis → a H. stretch by a factor of $\frac{1}{\frac{3}{4}} = \frac{4}{3}$ → SECOND

$$(x, y) \rightarrow \left(-\frac{4}{3}x, -\frac{2}{3}y\right)$$

f) $3y - 6 = f(\underbrace{-2x + 12})$

we need to rearrange these as brackets so that x has no coefficient inside the brackets, so that we can say that the stretch goes first.

factor (-2) out:

$$-2x + 12 = -2\left(\frac{-2x}{-2} + \frac{12}{-2}\right)$$

$$= -2(x - 6)$$

$$\therefore \quad 3y - 6 = f(-2(x-6))$$

$$3y = f(-2(x-6)) + 6 \quad \Big| \div 3 \text{ both sides}$$

$$y = \frac{1}{3}f(-2(x-6)) + \frac{6}{3}$$

$$y = \frac{1}{3}f(-2(x-6)) + 2$$

→ H. Shift Right by 6 units

V. stretch   H. reflection

shift right by 6 units

V. stretch by a factor of $\frac{1}{3}$

$\boxed{\text{THIRD}}$

H. reflection into the y-axis

$\boxed{\text{FIRST}}$

H. stretch by a factor of $\frac{1}{2}$

$\boxed{\text{SECOND}}$