p 040 q8

8. Given the function $y = f(x)$, write the equation of the form $y - k = af(b(x - h))$ that would result from each combination of transformations.

a) a vertical stretch about the x-axis by a factor of 3, a reflection in the x-axis, a horizontal translation of 4 units to the left, and a vertical translation of 5 units down

b) a horizontal stretch about the y-axis by a factor of $\frac{1}{3}$, a vertical stretch about the x-axis by a factor of $\frac{3}{4}$, a reflection in both the x-axis and the y-axis, and a translation of 6 units to the right and 2 units up

a) V. str. by a factor of 3 fits the coefficient a.

$\boxed{a=3}$

Refl. into the x-axis is a Vertical reflection & fits coefficient b => $\boxed{b=-1}$

a H. shift (translation) 4 units left fits the parameter h, & h must be negative so tha the move is to the left.

$\boxed{h=-4}$

a V. translation (shift) 5 units down means: $k=-5$ when k is on the right like so:

$y = \text{~~~} + k$

$$y = \sim\sim\sim + \underbrace{k}_{= -5}$$

$$y = \sim\sim\sim + (-5)$$

but if we move k to the other side of the equality:

$$y - k = \sim\sim\sim$$

$$y - (-5) = \sim\sim\sim$$

k still has to be equal to (-5).

$$\therefore \quad \boxed{k = -5}$$

Let's glue all of these together

$$y - (-5) = 3f(-(x - (-4)))$$

$$y + 5 = 3f(-(x + 4))$$

b) H. str. by a factor of $\frac{1}{3}$ means

$$b = \frac{1}{\frac{1}{3}}$$ ← b/c the

$\frac{1}{3}$ ← b/c the function definition and the mapping have reciprocal factors, and the H. stretch factor is always taken from the mapping rather than the function definition.

$$b = 1 \times \frac{3}{1} = 3$$

$$\boxed{b = 3}$$

V. str. by a factor of $\frac{3}{4}$ is the same as the coefficient a.

$$\boxed{a = \frac{3}{4}}$$

Reflections in both H & V. means that both the coefficient a and b must