

12. Alison often sketches her quilt designs on a coordinate grid. The coordinates for a section of one her designs are $A(-4, 6)$, $B(-2, -2)$, $C(0, 0)$, $D(1, -1)$, and $E(3, 6)$. She wants to transform the original design by a horizontal stretch about the y -axis by a factor of 2, a reflection in the x -axis, and a translation of 4 units up and 3 units to the left.

- Determine the coordinates of the image points, A' , B' , C' , D' , and E' .
- If the original design was defined by the function $y = f(x)$, determine the equation of the design resulting from the transformations.

a) It is easiest to work w/ the mapping.
The transformation is

1) H. str. by a factor of 2

$$(x, y) \rightarrow (2x, y)$$

2) A reflection into the x -axis
is a vertical one

$$\Rightarrow (x, y) \rightarrow (x, -y)$$

3) 4 units up $\Rightarrow k = 4 \Rightarrow (x, y) \rightarrow (x, y + 4)$
3 units left $\Rightarrow h = 3 \Rightarrow (x, y) \rightarrow (x - 3, y)$

If we glue it all together we get

$$(x, y) \rightarrow (2x - 3, -y + 4)$$

Let's build a table that helps calculate the image points:

	x, y	$2x - 3, -y + 4$
A	-4, 6	$-8 - 3, -6 + 4 = -11, -2 = A'$

A	-4, 6	$-8-3, -6+4 = -11, -2 = A'$
B	-2, -2	$2(-2)-3, -(-2)+4 = -7, 6 = B'$
C	0, 0	$2 \cdot 0 - 3, -0 + 4 = -3, 4 = C'$
D	1, -1	$2(1)-3, (-)(-1)+4 = -1, 5 = D'$
E	3, 6	$2(3)-3, -6+4 = 3, -2 = E'$

b) The equation is the function formula of $g(x)$
 from a: $(x, y) \rightarrow (2x-3, -y+4)$
 using traditional order of stretch first and
 horizontal first we get

$$g(x) = a \cdot f(b(x-h)) + k$$

$$(x, y) \rightarrow (2x-3, -y+4)$$

$\rightarrow a = -1$

$\rightarrow k = 4$

↓
1st
||
 $b = \frac{1}{2}$

↓
becomes
 $h = -3$

↓
 $(x - (-3)) = (x+3)$
LEFT

$$\therefore \boxed{g(x) = -f\left(\frac{1}{2}(x+3)\right) + 4}$$