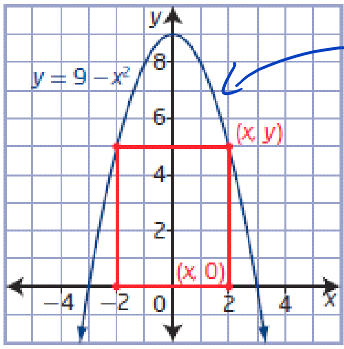
p 041 q16

16. A rectangle is inscribed between the x-axis and the parabola $y = 9 - x^2$ with one side along the x-axis, as shown.

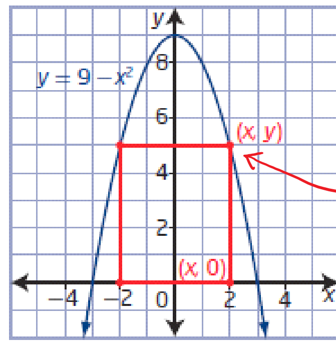


let's call this function $f(x)$

a) Write the equation for the area of the rectangle as a function of x.

b) Suppose a horizontal stretch by a factor of 4 is applied to the parabola. What is the equation for the area of the transformed rectangle?

c) Suppose the point (2, 5) is the vertex of the rectangle on the original parabola. Use this point to verify your equations from parts a) and b).



these are also known as vertices.

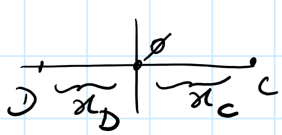
attention!

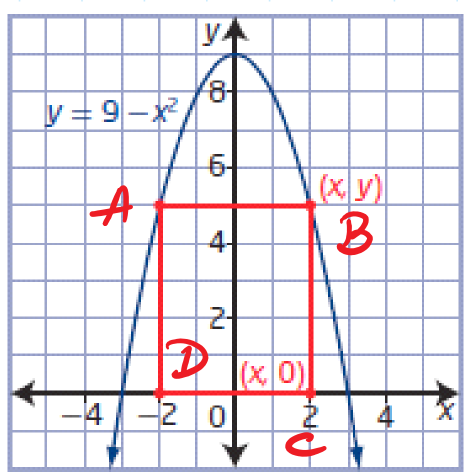
do not confuse the vertex of a polygon such as the rectangle w/ the vertex of the parabola

a) Let's give each corner of the rectangle a letter: A, B, C & D.

To calculate the area of the rectangle ABCD we want to know the edges: DC and BC.

The length of DC is made of $2 \cdot x_C$ b/c $-x_D = x_C$





$DC = 2x_c$ (*)

The length of BC: is the same as y_B.

We know that B is on the parabola $f(x)$ $\Rightarrow$

$$f(x_B) = y_B$$

← this gives us a way to calculate y_B, which gives us BC.

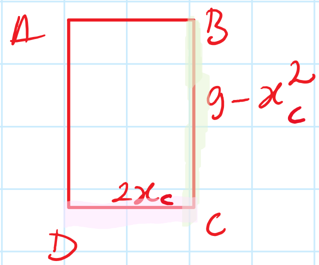
$\Downarrow$

$$y_B = f(x_B) = 9 - x_B^2$$

we can see on the graph: $x_B = x_c$

$$\Rightarrow \quad y_B = 9 - x_c^2 \qquad (**)$$

So far we have



from (**)

from (*)

$$\text{Area} = DC \times BC = (2x_c)(9 - x_c^2)$$

← the only variable is x_c so we can drop the subscript c.

$$\Rightarrow \boxed{\text{Area}(x) = (2x)(9 - x^2)}$$

so this is now a function of x.

so this is then a function of x.

b) the area above can also be said to be

$$A(x) = (2x)(9-x^2)$$

$$\boxed{A(x) = (2x)\,f(x)} \quad (*)$$

b/c $f(x) = 9 - x^2$

b) $f(x) = 9 - x^2$ = the parabola

← A Horiz. stretch by a factor of 4 has the reciprical factor in the mapping of the transformation

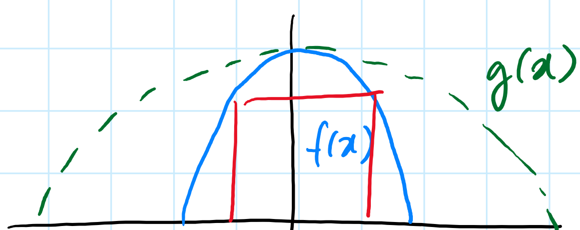
$$4 \rightarrow \frac{1}{4}$$

The stretched parabola is:

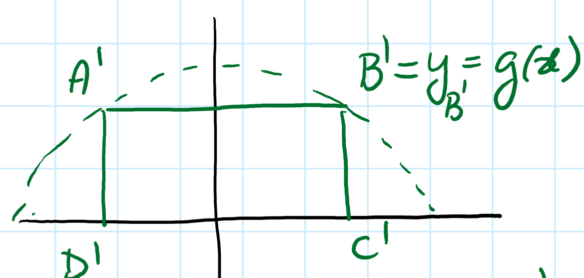
$$g(x) = f\left(\tfrac{1}{4}(x)\right) = 9 - \left(\tfrac{1}{4}x\right)^2 = 9 - \left(\tfrac{1}{4}\right)^2 x^2 = 9 - \frac{1}{16}x^2$$

$$\boxed{g(x) = 9 - \frac{1}{16}x^2} \quad (**)$$

We want to replaced $g(x)$ in the formula at $(*)$



Area$(x) = 2x \cdot f(x)$



Area$(x) = 2x \cdot g(x)$

$g(x) = 9 - \frac{1}{16}x^2 \rightarrow$ from $(**)$

$g(x) = 9 - \frac{1}{16}x^2 \rightarrow$ from (x,y)

Therefore the area is now:

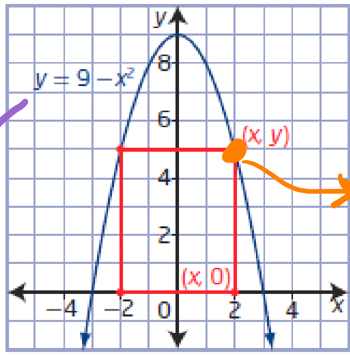
$$Area(x) = 2x\left(9 - \frac{1}{16}x^2\right) = 2 \cdot 9x - \cancel{2} \cdot x \cdot \frac{1}{\cancel{16}_{8}} x^2$$

$$= 18x - \frac{1}{8}x^3$$

$$\boxed{Area(x) = -\frac{1}{8}x^3 + 18x}$$

(c). Does point (2,5) verify to be present on the original parabola?

f(x)



point (2,5) is on the parabola

We verify so:

is $f(2) = 5$?

$$9 - x^2 \stackrel{?}{=} 5$$

$$9 - 2^2 = 5$$

$$9 - 4 = 5 \checkmark$$

This verifies that (2,5) is indeed on $f(x)$.

$g(x)$ = the transformation in (b)

$$= f\left(\tfrac{1}{4}x\right)$$

The point (2,5) was stretched horizontally by a factor of 4

This means based on the mapping $(x, y) \rightarrow (4x, y)$

$$\Rightarrow x_g = 4 \cdot x_f$$

$\Downarrow$

$x_g = 8$

$\Rightarrow (2,5) \rightarrow (8,5)$

The y_g did not change since the transformation was not vertical in nature

$\Rightarrow$ we must verify $g(x_g) = y_g = f(\frac{1}{4}x) = 9 - (\frac{1}{4}x)^2$

8 5

$5 \stackrel{?}{=} g(8)$

$9 - \frac{1}{16}x^2 \stackrel{?}{=} 5$

$9 - \frac{1}{16} \cdot 64 \stackrel{?}{=} 5$

$9 - 4 = 5$ ✓

$\therefore$ it all verifies