p 073 q5(a)

5. Sketch the graph of each function using transformations. State the domain and range of each function.

a) $f(x) = \sqrt{-x} - 3$

b) $r(x) = 3\sqrt{x+1}$

c) $p(x) = -\sqrt{x-2}$

d) $y - 1 = -\sqrt{-4(x-2)}$

e) $m(x) = \sqrt{\frac{1}{2}x} + 4$

f) $y + 1 = \frac{1}{3}\sqrt{-(x+2)}$

a) As always our parent baseline function is the square root radical:

$$p(x) = \sqrt{x}$$

which looks like so:



w/ an original domain of $D_p = \{x \mid x \geq 0, x \in \mathbb{R}\}$

and a range of $R_p = \{y \mid y \geq 0, y \in \mathbb{R}\}$

The given function

$$f(x) = \sqrt{-x} - 3$$

can be written as a transformation of $p(x)$ like so:

$$\sqrt{-x} = p(-x)$$

$$f(x) = \sqrt{-x} - 3 = p(-x) - 3$$

$p(-x)$: this is a horizontal stretch by a factor of -1, which is actually a horiz. reflection into the y-axis

-3: this is a vertical shift down by 3 units

this is a horizontal only stretch (reflection in this case)

this is a vertical only shift.

only stretch (reflection in this case.

vertical only shift.

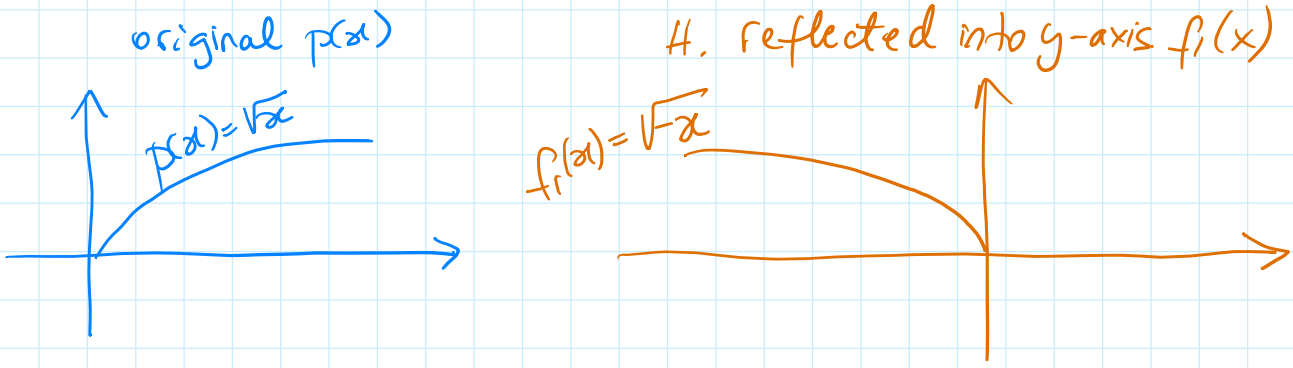
Theory when you have a combination of two transformations of which one is horizontal only and the other is vertical only then the order in which these two are applied does not matter.

Proof: https://www.desmos.com/calculator/t4rbmw5wwu

Now that we identifies a
- reflection
- shift

it's easy to sketch

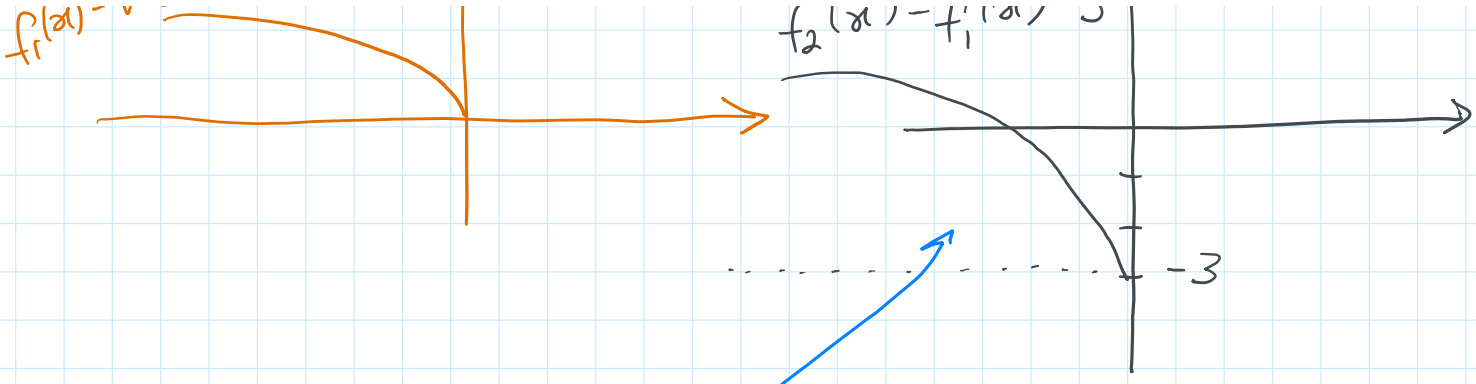
1) let's start w/ the reflection: it's horizontal into the y-axis



2) next we apply the vertical shift, on the last obtained function $f_1(x)$

i.e. $f_2(x) = f_1(x) - 3$



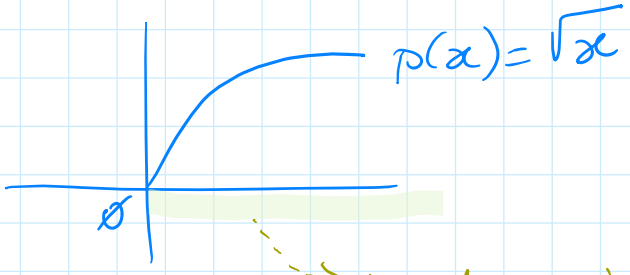


That is we drag the function downwards, vertically by 3 units

This is the final result of the sketch ble $f_2(x)$ is actually equal to $f(x)$.

What about the domain and the range?

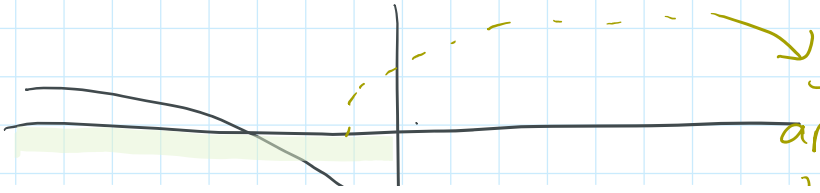
First let's look at the original domains:



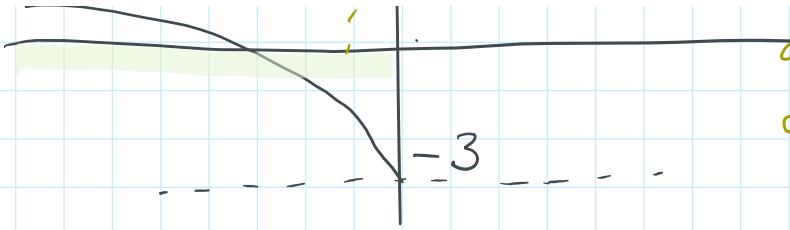
→ the domain is always to the right of zero:

$x \geq 0$

In our result



→ the x-s in this domain are always to the left of



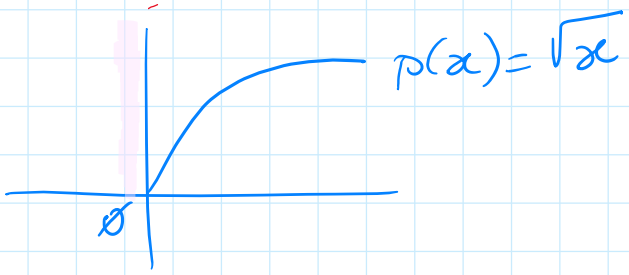
are always to the left of zero

$x \leq 0$

written nicely this becomes

The answer for the domain: $D_f = \{x \mid x \leq 0 \; ; \; x \in \mathbb{R}\}$.

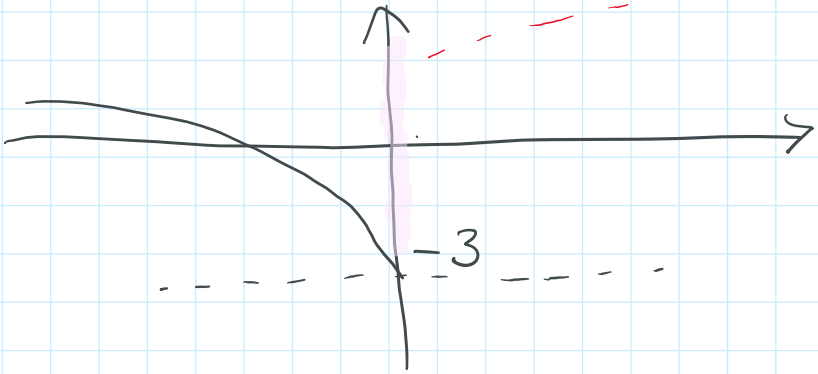
Let's look at the range of our original $p(x)$



→ the y-s are always larger (or equal) than zero

$y \geq 0$

Then similarly in our result



→ here the y-s are always larger (or equal) than -3

$y \geq -3$

Written nicely, the answer for the Range is: $R_f = \{y \mid y \geq -3, y \in \mathbb{R}\}$