p073, q9

Friday, November 25, 2022 5:11 PM

9. The function $4 - y = \sqrt{3x}$ is translated 9 units up and reflected in the x-axis.

a) Without graphing, determine the domain and range of the image function.

b) Compared to the base function, $y = \sqrt{x}$, by how many units and in which direction has the given function been translated horizontally? vertically?

a) $4 - y = \sqrt{3x}$

$-y = \sqrt{3x} - 4$

$y = -\sqrt{3x} + 4$

↑ this is going to be our parent baseline function that we are acting on.

$f(x) = -\sqrt{3x} + 4$

The image function in this context means the transformation, let's call it $g(x)$

there are two steps:
1) translate 9 units up → $g_1(x)$
2) reflect into the x-axis → $g_2(x)$

Therefore:

$g_1(x) = f(x) + 9$

$g_2(x) = -g_1(x)$

$g_2(x) = -g_1(x)$

↓

this is going to be our final $g(x)$

$$g(x) = g_2(x) = -g_1(x) = -[f(x) + 9]$$

↑ we calculate $f(x)$ at (x)

$$f(x) = -\sqrt{3x} + 4$$

$$\therefore\ g(x) = -[-\sqrt{3x} + 4 + 9]$$

$$g(x) = -[-\sqrt{3x} + 13]$$

$$g(x) = \sqrt{3x} - 13$$ ← this is our image function and the parent was

To find the domain & range of $g(x)$ we could compare to $f(x)$, but an even better comparison would be to compare to the ultimate baseline function $p(x) = \sqrt{x}$

which has $D_p = \{x \geq 0\}$

$R_p = \{y \geq 0\}$.

IN RELATION TO $p(x)$:

we can see that $g(x) = \sqrt{3x} - 13$ was stretched horizontally by a factor of $\frac{1}{3}$ → this does not change the domain or the range of the original $p(x)$ but the second step $g(x) = \sqrt{3x} - 13$ (2nd step) does shift the entire graph down by 13 units, so the range will be affected as $y \geq -13$

$$\therefore D_g = \{x, x \geq 0, x \in \mathbb{R}\}$$
$$R_g = \{y, y \geq -13, y \in \mathbb{R}\}.$$

b) we have the relation between $g(x)$ and the base parent $p(x) = \sqrt{x}$

$g(x) = \sqrt{3x} - 13$

step 1 : we have a horiz. stretch rather than a horiz translation so this part of the answer is

Answer: that the function has not been translated horizontally at all, in relation to $\sqrt{x}$

step 2: -13

Answer: this is a vertical translation by 13 units downwards in relation to $\sqrt{x}$