p 218 q7

Monday, October 13, 2025 1:28 PM

7. a) What is the equation for any circle with centre at the origin and radius 1 unit?

b) Determine the value(s) for the missing coordinate for all points on the unit circle satisfying the given conditions. Draw diagrams.

i) $\left(\frac{2\sqrt{3}}{5}, y\right)$

ii) $\left(x, \frac{\sqrt{7}}{4}\right), x < 0$

c) Explain how to use the equation for the unit circle to find the value of cos θ if you know the y-coordinate of the point where the terminal arm of an angle θ in standard position intersects the unit circle.

a) the general form of such equations are x^2 + y^2 = r^2
and since the radius is r = 1 we can replace this with

$$x^2 + y^2 = 1^2$$

or rather

$$x^2 + y^2 = 1$$

↑ this is called the unit circle

b) i) $\left(\frac{2\sqrt{3}}{5}, y\right)$ since this point is on the unit circle it must follow the equation of the unit circle which is $x^2 + y^2 = 1$

$\left(\frac{2\sqrt{3}}{5}, y\right) \Rightarrow \begin{cases} x = \frac{2\sqrt{3}}{5} \\ \text{and} \\ y \text{ is just } y \text{ stays as is} \end{cases} \Rightarrow$

$$\Rightarrow \left(\frac{2\sqrt{3}}{5}\right)^2 + y^2 = 1$$

from here we can calculate y

$$y^2 = 1 - \left(\frac{2\sqrt{3}}{5}\right)^2$$

$$y^2 = \overset{25}{1} - \left(\frac{4 \cdot 3}{25}\right)$$

$$y^2 = \frac{1 \cdot 25}{1 \cdot 25} - \left(\frac{12}{25}\right) = \frac{25 - 12}{25} = \frac{13}{25}$$

$$y^2 = \frac{13}{25}$$

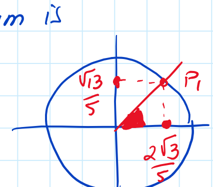
$$y = \pm\sqrt{\frac{13}{25}}$$

$$y = \pm\frac{\sqrt{13}}{5}$$

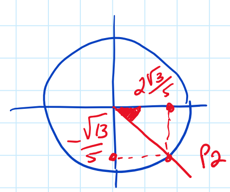
the two possible points are $\left(\frac{2\sqrt{3}}{5}, \frac{\sqrt{13}}{5}\right) = P_1$

and $\left(\frac{2\sqrt{3}}{5}, -\frac{\sqrt{13}}{5}\right) = P_2$

the diagram is



and



the two angles are equal as marked in red and are symmetric in relation to the x-axis

(not equal in standard position though)

(ii) $\left(x, \frac{\sqrt{7}}{4}\right)$ and $x<0$.

again since the point is on the unit circle it must fullfill the relation:

$$x^2+y^2=1$$

for this point then:

$$x^2+\left(\frac{\sqrt{7}}{4}\right)^2=1 \quad \text{and } x<0$$

Solve for x:

$$x^2+\left(\frac{7}{16}\right)=1 \Rightarrow x^2=\overset{16}{1}-\frac{7}{16}$$

$$x^2=\frac{1\cdot 16}{16}-\frac{7}{16}$$

$$x^2=\frac{16-7}{16}=\frac{9}{16}=\frac{3^2}{4^2}=\left(\frac{3}{4}\right)^2$$

$$x^2=\left(\frac{3}{4}\right)^2$$

$$x=\pm\sqrt{\left(\frac{3}{4}\right)^2}=\pm\frac{3}{4}$$

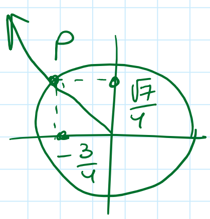
we are requested to only have $x<0$

$\Downarrow$

the only answer is $x=-\frac{3}{4}$

Thus there is only one point that satisfies $\left(x, \frac{\sqrt{7}}{4}\right)$ so:

$\left(-\frac{3}{4}, \frac{\sqrt{7}}{4}\right)$ and the diagram is



I disagree w/ the second angle in Q, in the diagram in the answer key. We were not asked to resolve $\left(-x, \frac{\sqrt{7}}{4}\right)$ which is also on the unit circle and would conviniently support a negative x as well. We were only asked for $\left(x, \frac{\sqrt{7}}{4}\right)$ with $x<0$ which cannot fit anything but a Q2 solution.

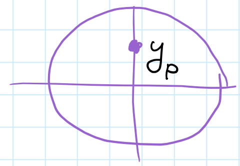
c)

c) Explain how to use the equation for the unit circle to find the value of cos θ if you know the y-coordinate of the point where the terminal arm of an angle θ in standard position intersects the unit circle.

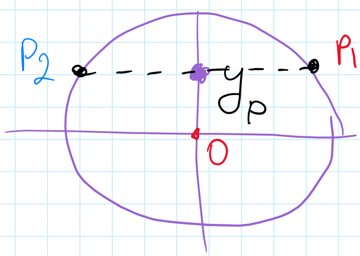
step1 Let's visualize this 1st:

we know the y-coordinate

let's say it's positive. as here

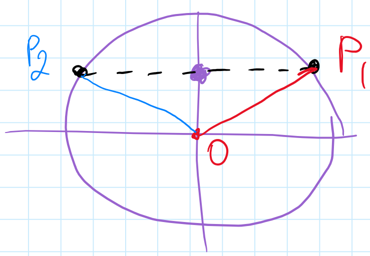


Step2 where does this y-coordinate intersect the unit circle?



the answer is in 2 points P_1 & P_2, one on the left, the other on the right.

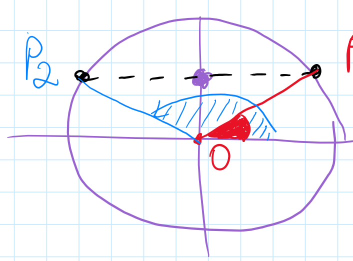
step3 this determines 2 possible terminal arms: OP_1 & OP_2



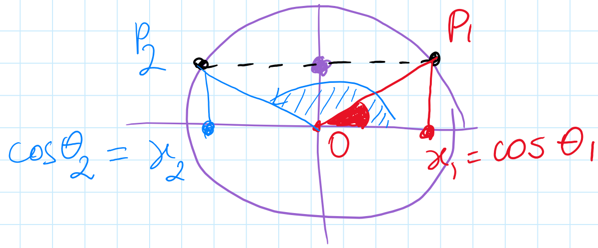
and their two angles:

θ_1 & θ_2

OP_1 OP_2



their $\cos\theta_{1 \text{ or } 2}$ are the x-coordinates of P_1 & P_2



<u>Step 4</u>

using the unit circle equation to find $\cos\theta$ when $y\,(=y_p)$ is known.

P: (or rather P_1 & P_2) are on the circle

$$\Rightarrow \quad x_p^2 + y_p^2 = 1$$

$\cos\theta = x_p$ and solving for x_p is as follows

$$x_p^2 = 1 - y_p^2$$

$$x_p = \pm\sqrt{1-y_p^2}$$

$$-\sqrt{1-y_p^2} = x_{p_1}$$

$$+\sqrt{1-y_p^2} = x_{p_2}$$

therefore the two solutions for $\cos\theta = x_p$ are:

$$\boxed{\cos\theta_{1,2} = \pm\sqrt{1-y_p^2}}$$

where y_p is known but it is also known that the y-coord. is the same as the $\sin\theta$

so this solution can also be stated as

$$\boxed{\cos\theta_{1,2} = \pm\sqrt{1-\sin^2\theta}}$$