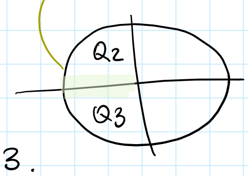
8. Suppose that the cosine of an angle is negative and that you found one solution in quadrant III.

a) Explain how to find the other solution between 0 and 2π.

b) Describe how to write the general solution.

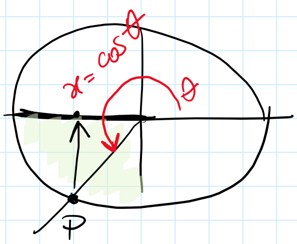
$\cos\theta = x =$ negative



all angles in Q_2 & Q3 can cast an x that is negative and x = cosine

the cosine is negative and we are told we are in Q3.

a)

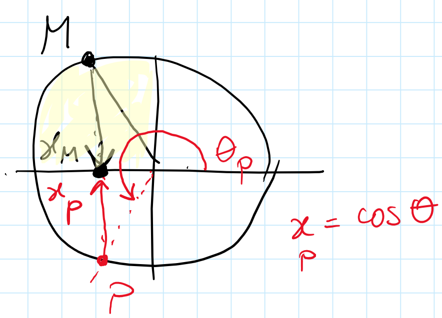


the other solution in $[0, 2\pi]$ is on the same $x = \cos\theta$

the other point (let's call it M) will project its x from above the x-axis like so

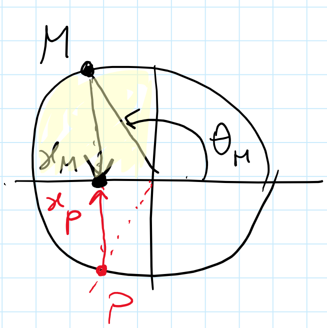
M then also has $\cos\theta$

b/c $x_M = x_P = \cos\theta$

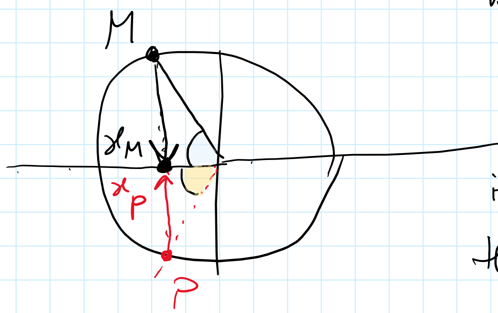


since it projects the x from above the x-axis we conclude M is in Q_2.

to answer (b) we have to see a relationship between θ_P and θ_M



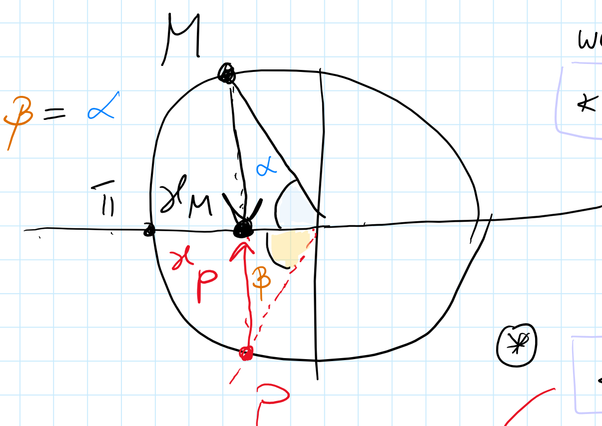
symmetry gives us the following equality symmetry gives us the following equality between the blue angle & the orange angle.



it's the only way that x_M can equal x_P

let's name them α & β.

$\beta = \alpha$



we can see then that

$$\boxed{\measuredangle P = \theta = \pi + \beta}$$

and angle M does not reach π and is α away from π, thus

(*) $$\boxed{\measuredangle M = \pi - \alpha}$$

one way to answer this is to say that we must subtract the reference angle (α) from π. (this is the answer in the answer key)

and $\alpha = \beta$

and $\beta = \theta - \pi$
$\theta \downarrow \measuredangle P$

b) let's construct a general solution that includes both M and P.
a general solution has ∞ rotations.
we start counterclockwise and encounter M first then we move on by $\alpha + \beta = \alpha + \alpha = 2\alpha$ and so arrive at P.
($\uparrow$ b/c $\alpha = \beta$)

To get to M repeatedly we need a full circle rotation from M, i.e.
$\measuredangle M + 2\pi$

To get to M repeatedly we need a full circle rotation from M, i.e.

$\measuredangle M + 2\pi$

and w/ ∞ rotations that's

$\measuredangle M + 2\pi n, n \in \mathbb{I}$

n is ⊕ or ⊖ b/c u can move either clockwise or counterclockwise.

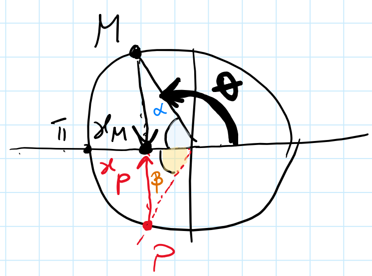
To get to P repeatedly it's the same

$\measuredangle P + 2\pi n, n \in \mathbb{I}$

The solution set is $\{\hat{M} + 2\pi n, n \in \mathbb{I}\} \cup \{\hat{P} + 2\pi n, n \in \mathbb{I}\}$

if this time we name $\hat{M}$ to be θ instead of $\hat{P}$ the first part becomes $\{\theta + 2\pi n, n \in \mathbb{I}\}$

and for the 2nd part we want to calculate $\hat{P}$:



$$\left.\begin{matrix}\theta + \alpha = \pi \\ \alpha = \beta\end{matrix}\right\} \Rightarrow$$

$\Rightarrow \theta + \beta = \pi$

isolate β:

$\boxed{\beta = \pi - \underset{\text{or } \hat{M}}{\theta}}$

from the diagram $\hat{P} = \pi + \beta = \pi + (\pi - \theta) = \pi + \pi - \theta = 2\pi - \theta$

∴ the general solution is

$$\{\theta + 2\pi n, n \in \mathbb{I}\} \cup \{\underbrace{2\pi - \theta + 2\pi n}_{2\pi(1+n) - \theta}, n \in \mathbb{I}\}$$

$2\pi(n+1) - \theta$

$= \Big\{\{\theta + 2\pi n\} \cup \{2\pi(n+1) - \theta\}$ so that $n \in \mathbb{I}$ is applied in pairs$\Big\}$

this is a more specific solution than saying

$\{\theta_1 + 2\pi n, n \in \mathbb{I}\} \cup \{\theta_2 + 2\pi n, n \in \mathbb{I}\}$ suggested by the answer key.