

$$2 \cos^2 \theta + 3 \cos \theta - 2 = 0$$

replace : $x := \cos \theta$

↓

$$2x^2 + 3x - 2 = 0$$

factorize - square method

$$\begin{array}{lcl} 2x & \overset{\ominus}{-} & 1 \rightarrow x \\ x & \underset{\oplus}{+} & 2 \rightarrow 4x \end{array} \left. \vphantom{\begin{array}{lcl} 2x & \overset{\ominus}{-} & 1 \\ x & \underset{\oplus}{+} & 2 \end{array}} \right\} \begin{array}{l} \text{can combine} \\ 4x - x = 3x \\ \downarrow \quad \downarrow \\ \oplus \quad \ominus \end{array}$$

Attempt: $(2x-1)(x+2)$

verify $2x^2 + 4x - x - 2 = 2x^2 + 3x - 2$ ✓

Thus $2x^2 + 3x - 2 = 0$
 becomes $(2x-1)(x+2) = 0$

when $a \times b = 0$
 there are two solutions;
 $a = 0$
 and $b = 0$
 both provide a 0 result
 to the equation.

solutions: i) $2x-1=0$
 $2x=1$
 $x = 1/2$

ii) $x+2=0$

$$\text{II) } x+2=0$$

$$x=-2$$

Restore x , it was

$$x := \cos \theta$$

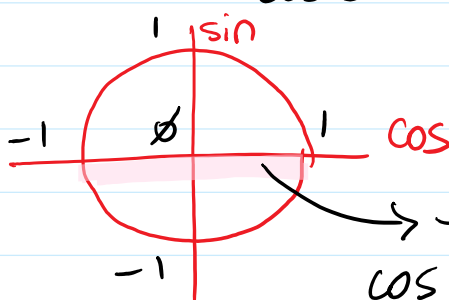
now we can say

$$\cos \theta = \text{either } \frac{1}{2} \text{ or } -2.$$

not a
trigonometrical
solution, we

can discard it, no angle

$$\text{can } \cos \theta = -2$$



the range of
 $\cos$ is defined in
 $[-1, 1]$

The only solution then remains ;

$$\cos \theta = \frac{1}{2}$$

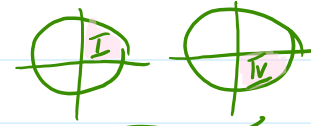
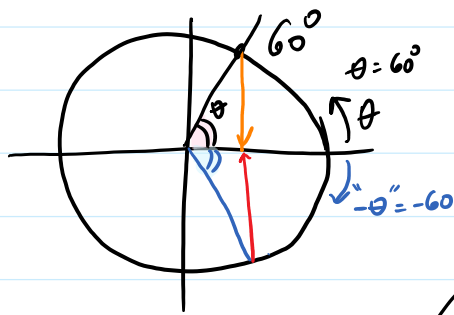
INVERSE IT, to find the angle

$$\cos^{-1}\left(\frac{1}{2}\right) = \theta$$

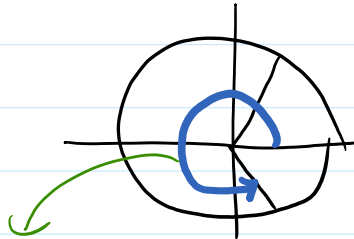
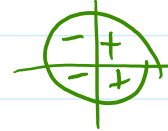
use your calculator:

$$\theta = 60^\circ$$

what matching angles have the same $\cos$ (or x) values?



$\cos$ is positive in these quadrants & it's negative in the others:



the no-negative, proper rotation angle is $360^\circ - 60^\circ = 300^\circ$.

Thus there are 2 solutions to the given equation for one rotation: $0 \leq \theta \leq 360^\circ$
 $\{60^\circ, 300^\circ\}$